Chapter 1

Foundations for Geometry

1A Euclidean and Construction Tools

1-1 Understanding Points, Lines, and Planes
Lab Explore Properties Associated with Points
1-2 Measuring and Constructing Segments
1-3 Measuring and Constructing Angles
1-4 Pairs of Angles

1B Coordinate and Transformation Tools

1-5 Using Formulas in Geometry
1-6 Midpoint and Distance in the Coordinate Plane
1-7 Transformations in the Coordinate Plane
Lab Explore Transformations

Picture This!

Many geometric concepts and shapes may be used in creating works of art. Unique designs can be made using only points, lines, planes, or circles.
**Vocabulary**

Match each term on the left with a definition on the right.

1. coordinate  
   A. a mathematical phrase that contains operations, numbers, and/or variables
2. metric system of measurement  
   B. the measurement system often used in the United States
3. expression  
   C. one of the numbers of an ordered pair that locates a point on a coordinate graph
4. order of operations  
   D. a list of rules for evaluating expressions
5. decimal system of weights and measures that is used universally in science and commonly throughout the world

**Measure with Customary and Metric Units**

For each object tell which is the better measurement.

5. length of an unsharpened pencil 7 1/2 in. or 9 1/4 in.
6. the diameter of a quarter 1 m or 2 1/2 cm
7. length of a soccer field 100 yd or 40 yd
8. height of a classroom 5 ft or 10 ft
9. height of a student’s desk 30 in. or 4 ft
10. length of a dollar bill 15.6 cm or 35.5 cm

**Combine Like Terms**

Simplify each expression.

11. \(-y + 3y - 6y + 12y\)  
12. \(63 + 2x - 7 - 4x\)
13. \(-5 - 9 - 7x + 6x\)  
14. \(24 - 3y + y + 7\)

**Evaluate Expressions**

Evaluate each expression for the given value of the variable.

15. \(x + 3x + 7x\) for \(x = -5\)  
16. \(5p + 10\) for \(p = 78\)
17. \(2a - 8a\) for \(a = 12\)  
18. \(3n - 3\) for \(n = 16\)

**Ordered Pairs**

Write the ordered pair for each point.

19. \(A\)  
20. \(B\)
21. \(C\)  
22. \(D\)
23. \(E\)  
24. \(F\)
Previously, you

- used the order of operations.
- used variables and expressions to represent situations.
- located points in the coordinate plane.
- solved equations.

In This Chapter

You will study

- applying basic facts about points, lines, planes, segments, and angles.
- measuring and constructing segments and angles.
- using formulas to find distance and the coordinates of a midpoint.
- identifying reflections, rotations, and translations.

Where You’re Going

You can use the skills learned in this chapter

- to find distances between cities.
- to determine how much material is needed to make a rectangular or triangular object.
- in classes such as Biology, when you learn about gene mapping and in physics, when you study angles formed by light waves that bounce off objects.

Key Vocabulary/Vocabulario

<table>
<thead>
<tr>
<th>Term</th>
<th>Spanish Term</th>
</tr>
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<tbody>
<tr>
<td>angle</td>
<td>ángulo</td>
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<tr>
<td>area</td>
<td>área</td>
</tr>
<tr>
<td>coordinate plane</td>
<td>plano cartesiano</td>
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<tr>
<td>line</td>
<td>línea</td>
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<td>perimeter</td>
<td>perímetro</td>
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<td>plane</td>
<td>plano</td>
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<tr>
<td>point</td>
<td>punto</td>
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<tr>
<td>transformation</td>
<td>transformación</td>
</tr>
<tr>
<td>undefined term</td>
<td>término indefinido</td>
</tr>
</tbody>
</table>

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. A **definition** is a statement that gives the meaning of a word or phrase. What do you think the phrase **undefined term** means?

2. **Coordinates** are numbers used to describe a location. A **plane** is a flat surface. How can you use these meanings to understand the term **coordinate plane**?

3. A **point** is often represented by a dot. What real-world items could represent points?

4. **Trans-** is a prefix that means “across,” as in movement. A **form** is a shape. How can you use these meanings to understand the term **transformation**?
Reading Strategy: Use Your Book for Success

Understanding how your textbook is organized will help you locate and use helpful information.

As you read through an example problem, pay attention to the notes in the margin. These notes highlight key information about the concept and will help you to avoid common mistakes.

The Glossary is found in the back of your textbook. Use it when you need a definition of an unfamiliar word or phrase.

The Index is located at the end of your textbook. If you need to locate the page where a particular concept is explained, use the Index to find the corresponding page number.

The Skills Bank is located in the back of your textbook. Look in the Skills Bank for help with math topics that were taught in previous courses, such as the order of operations.

Try This

Use your textbook for the following problems.

1. Use the index to find the page where right angle is defined.
2. What formula does the Know-It Note on the first page of Lesson 1-6 refer to?
3. Use the glossary to find the definition of congruent segments.
4. In what part of the textbook can you find help for solving equations?
The most basic figures in geometry are **undefined terms**, which cannot be defined by using other figures. The undefined terms **point**, **line**, and **plane** are the building blocks of geometry.

### Undefined Terms

<table>
<thead>
<tr>
<th>TERM</th>
<th>NAME</th>
<th>DIAGRAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>A point</td>
<td>names a location and has no size. It is represented by a dot.</td>
<td>A capital letter point $P$</td>
</tr>
<tr>
<td>A line</td>
<td>is a straight path that has no thickness and extends forever.</td>
<td>A lowercase letter or two points on the line $\ell$, $XY$ or $YX$</td>
</tr>
<tr>
<td>A plane</td>
<td>is a flat surface that has no thickness and extends forever.</td>
<td>A script capital letter or three points not on a line plane $\mathcal{R}$ or plane $ABC$</td>
</tr>
</tbody>
</table>

Points that lie on the same line are **collinear**. $K, L,$ and $M$ are collinear. $K, L,$ and $N$ are **noncollinear**. Points that lie in the same plane are **coplanar**. Otherwise they are **noncoplanar**.

### Example 1

**Naming Points, Lines, and Planes**

Refer to the design in the roof of Beijing’s National Stadium.

**A** Name four coplanar points. $K, L, M,$ and $N$ all lie in plane $\mathcal{R}$.

**B** Name three lines. $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.

1. Use the diagram to name two planes.
Segments and Rays

DEFINITION | NAME | DIAGRAM
--- | --- | ---
A segment, or line segment, is the part of a line consisting of two points and all points between them. | The two endpoints \( \overrightarrow{AB} \) or \( \overrightarrow{BA} \) | \( A \) and \( B \)
An endpoint is a point at one end of a segment or the starting point of a ray. | A capital letter \( C \) and \( D \) | \( C \) and \( D \)
A ray is a part of a line that starts at an endpoint and extends forever in one direction. | Its endpoint and any other point on the ray \( \overrightarrow{RS} \) | \( R \) and \( S \)
Opposite rays are two rays that have a common endpoint and form a line. | The common endpoint and any other point on each ray \( \overrightarrow{EF} \) and \( \overrightarrow{EG} \) | \( F \), \( E \), and \( G \)

Example 2

Drawing Segments and Rays

Draw and label each of the following.

A a segment with endpoints \( U \) and \( V \)
B opposite rays with a common endpoint \( Q \)

2. Draw and label a ray with endpoint \( M \) that contains \( N \).

A postulate, or axiom, is a statement that is accepted as true without proof. Postulates about points, lines, and planes help describe geometric properties.

Postulates

1-1-1 Through any two points there is exactly one line.
1-1-2 Through any three noncollinear points there is exactly one plane containing them.
1-1-3 If two points lie in a plane, then the line containing those points lies in the plane.

Example 3

Identifying Points and Lines in a Plane

Name a line that passes through two points.

There is exactly one line \( n \) passing through \( G \) and \( H \).

3. Name a plane that contains three noncollinear points.
Recall that a system of equations is a set of two or more equations containing two or more of the same variables. The coordinates of the solution of the system satisfy all equations in the system. These coordinates also locate the point where all the graphs of the equations in the system intersect.

An intersection is the set of all points that two or more figures have in common. The next two postulates describe intersections involving lines and planes.

**Postulates Intersection of Lines and Planes**

1-1-4 If two lines intersect, then they intersect in exactly one point.

1-1-5 If two planes intersect, then they intersect in exactly one line.

Use a dashed line to show the hidden parts of any figure that you are drawing. A dashed line will indicate the part of the figure that is not seen.

**Example 4** Representing Intersections

Sketch a figure that shows each of the following.

- **A** A line intersects a plane, but does not lie in the plane.

- **B** Two planes intersect in one line.

**Think and Discuss**

1. Explain why any two points are collinear.
2. Which postulate explains the fact that two straight roads cannot cross each other more than once?
3. Explain why points and lines may be coplanar even when the plane containing them is not drawn.
4. Name all the possible lines, segments, and rays for the points A and B. Then give the maximum number of planes that can be determined by these points.
5. **Get Organized** Copy and complete the graphic organizer below. In each box, name, describe, and illustrate one of the undefined terms.
GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.
1. Give an example from your classroom of three collinear points.
2. Make use of the fact that endpoint is a compound of end and point and name the endpoint of $\overrightarrow{ST}$.

Use the figure to name each of the following.
3. five points
4. two lines
5. two planes
6. point on $\overrightarrow{BD}$

Draw and label each of the following.
7. a segment with endpoints $M$ and $N$
8. a ray with endpoint $F$ that passes through $G$

Use the figure to name each of the following.
9. a line that contains $A$ and $C$
10. a plane that contains $A$, $D$, and $C$

Sketch a figure that shows each of the following.
11. three coplanar lines that intersect in a common point
12. two lines that do not intersect

PRACTICE AND PROBLEM SOLVING

Use the figure to name each of the following.
13. three collinear points
14. four coplanar points
15. a plane containing $E$

Draw and label each of the following.
16. a line containing $X$ and $Y$
17. a pair of opposite rays that both contain $R$

Use the figure to name each of the following.
18. two points and a line that lie in plane $\mathcal{J}$
19. two planes that contain $\ell$

Sketch a figure that shows each of the following.
20. a line that intersects two nonintersecting planes
21. three coplanar lines that intersect in three different points
22. This problem will prepare you for the Multi-Step Test Prep on page 34. Name an object at the archaeological site shown that is represented by each of the following.
   a. a point
   b. a segment
   c. a plane

23. Draw each of the following.
24. \( \text{plane } \mathcal{H} \) containing two lines that intersect at \( M \)
25. \( \overrightarrow{ST} \) intersecting \( \mathcal{M} \) at \( R \)

25. Use the figure to name each of the following.
26. the intersection of \( \overrightarrow{TV} \) and \( \overrightarrow{US} \)
27. the intersection of \( \overrightarrow{US} \) and plane \( \mathcal{R} \)
28. the intersection of \( \overrightarrow{TU} \) and \( \overrightarrow{UV} \)

28. Write the postulate that justifies each statement.
29. The line connecting two dots on a sheet of paper lies on the same sheet of paper as the dots.
30. If two ants are walking in straight lines but in different directions, their paths cannot cross more than once.
31. Critical Thinking Is it possible to draw three points that are noncoplanar? Explain.

32. Tell whether each statement is sometimes, always, or never true. Support your answer with a sketch.
33. If two planes intersect, they intersect in a straight line.
34. If two lines intersect, they intersect at two different points.
35. \( \overline{AB} \) is another name for \( \overline{BA} \).
36. If two rays share a common endpoint, then they form a line.
37. Art Pointillism is a technique in which tiny dots of complementary colors are combined to form a picture. Which postulate ensures that a line connecting two of these points also lies in the plane containing the points?
38. Probability Three of the labeled points are chosen at random. What is the probability that they are collinear?
39. Campers often use a cooking stove with three legs. Which postulate explains why they might prefer this design to a stove that has four legs?
40. Write About It Explain why three coplanar lines may have zero, one, two, or three points of intersection. Support your answer with a sketch.
39. Which of the following is a set of noncollinear points?
   A  \( P, R, T \)  \( \quad \) C  \( P, Q, R \)
   B  \( Q, R, S \)  \( \quad \) D  \( S, T, U \)

40. What is the greatest number of intersection points four coplanar lines can have?
   F  6  \( \quad \) H  2
   G  4  \( \quad \) J  0

41. Two flat walls meet in the corner of a classroom. Which postulate best describes this situation?
   \( \Box \) Through any three noncollinear points there is exactly one plane.
   \( \Box \) If two points lie in a plane, then the line containing them lies in the plane.
   \( \Box \) If two lines intersect, then they intersect in exactly one point.
   \( \Box \) If two planes intersect, then they intersect in exactly one line.

42. Gridded Response  What is the greatest number of planes determined by four noncollinear points?

**CHALLENGE AND EXTEND**

Use the table for Exercises 43–45.

<table>
<thead>
<tr>
<th>Figure</th>
<th>( \quad )</th>
<th>( \quad )</th>
<th>( \quad )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Points</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Maximum Number of Segments</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

43. What is the maximum number of segments determined by 4 points?
44. **Multi-Step** Extend the table. What is the maximum number of segments determined by 10 points?
45. Write a formula for the maximum number of segments determined by \( n \) points.
46. **Critical Thinking** Explain how rescue teams could use two of the postulates from this lesson to locate a distress signal.

**SPIRAL REVIEW**

47. The combined age of a mother and her twin daughters is 58 years. The mother was 25 years old when the twins were born. Write and solve an equation to find the age of each of the three people. *(Previous course)*

Determine whether each set of ordered pairs is a function. *(Previous course)*
48. \( \{(0, 1), (1, -1), (5, -1), (-1, 2)\} \)
49. \( \{(3, 8), (10, 6), (9, 8), (10, -6)\} \)

Find the mean, median, and mode for each set of data. *(Previous course)*
50. 0, 6, 1, 3, 5, 2, 7, 10
51. 0.47, 0.44, 0.4, 0.46, 0.44
Explore Properties Associated with Points

The two endpoints of a segment determine its length. Other points on the segment are between the endpoints. Only one of these points is the midpoint of the segment. In this lab, you will use geometry software to measure lengths of segments and explore properties of points on segments.

**Activity**

1. Construct a segment and label its endpoints A and C.

2. Create point B on \(\overline{AC}\).

3. Measure the distances from A to B and from B to C. Use the Calculate tool to calculate the sum of \(AB\) and \(BC\).

4. Measure the length of \(\overline{AC}\). What do you notice about this length compared with the measurements found in Step 3?

5. Drag point B along \(\overline{AC}\). Drag one of the endpoints of \(\overline{AC}\). What relationships do you think are true about the three measurements?

6. Construct the midpoint of \(\overline{AC}\) and label it M.

7. Measure \(\overline{AM}\) and \(\overline{MC}\). What relationships do you think are true about the lengths of \(\overline{AC}, \overline{AM},\) and \(\overline{MC}\)? Use the Calculate tool to confirm your findings.

8. How many midpoints of \(\overline{AC}\) exist?

**Try This**

1. Repeat the activity with a new segment. Drag each of the points in your figure (the endpoints, the point on the segment, and the midpoint). Write down any relationships you observe about the measurements.

2. Create a point D not on \(\overline{AC}\). Measure \(\overline{AD}, \overline{DC},\) and \(\overline{AC}\). Does \(AD + DC = AC\)? What do you think has to be true about D for the relationship to always be true?
Objectives
Use length and midpoint of a segment.
Construct midpoints and congruent segments.

Vocabulary
coordinate
distance
length
congruent segments
construction
between
midpoint
bisect
segment bisector

Why learn this?
You can measure a segment to calculate the distance between two locations. Maps of a race are used to show the distance between stations on the course. (See Example 4.)

A ruler can be used to measure the distance between two points. A point corresponds to one and only one number on the ruler. This number is called a coordinate. The following postulate summarizes this concept.

Postulate 1-2-1 Ruler Postulate
The points on a line can be put into a one-to-one correspondence with the real numbers.

The distance between any two points is the absolute value of the difference of the coordinates. If the coordinates of points $A$ and $B$ are $a$ and $b$, then the distance between $A$ and $B$ is $|a - b|$ or $|b - a|$. The distance between $A$ and $B$ is also called the length of $\overline{AB}$, or $AB$.

Example 1 Finding the Length of a Segment
Find each length.

A $DC$
$DC = |4.5 - 2|$
$= |2.5|$
$= 2.5$

B $EF$
$EF = |-4 - (-1)|$
$= |-4 + 1|$
$= |-3|$
$= 3$

Find each length.
1a. $XY$
1b. $XZ$

Caution!
$PQ$ represents a number, while $\overline{PQ}$ represents a geometric figure. Be sure to use equality for numbers ($PQ = RS$) and congruence for figures ($\overline{PQ} \cong RS$).

Congruent segments are segments that have the same length. In the diagram, $PQ = RS$, so you can write $\overline{PQ} \cong \overline{RS}$. This is read as “segment $PQ$ is congruent to segment $RS$.” Tick marks are used in a figure to show congruent segments.
You can make a sketch or measure and draw a segment. These may not be exact. A **construction** is a way of creating a figure that is more precise. One way to make a geometric construction is to use a compass and straightedge.

**Construction** Congruent Segment

Construct a segment congruent to $\overline{AB}$.

**1.** Draw $\ell$. Choose a point on $\ell$ and label it $C$.

**2.** Open the compass to distance $AB$.

**3.** Place the point of the compass at $C$ and make an arc through $\ell$. Find the point where the arc and $\ell$ intersect and label it $D$.

$CD \cong AB$

**Example 2** Copying a Segment

Sketch, draw, and construct a segment congruent to $\overline{MN}$.

**Step 1** Estimate and sketch.
- Estimate the length of $\overline{MN}$ and sketch $\overline{PQ}$ approximately the same length.

**Step 2** Measure and draw.
- Use a ruler to measure $\overline{MN}$. $\overline{MN}$ appears to be 3.1 cm. Use a ruler and draw $\overline{XY}$ to have length 3.1 cm.

**Step 3** Construct and compare.
- Use a compass and straightedge to construct $\overline{ST}$ congruent to $\overline{MN}$.

A ruler shows that $\overline{PQ}$ and $\overline{XY}$ are approximately the same length as $\overline{MN}$, but $\overline{ST}$ is precisely the same length.

2. Sketch, draw, and construct a segment congruent to $\overline{JK}$.

In order for you to say that a point $B$ is **between** two points $A$ and $C$, all three of the points must lie on the same line, and $AB + BC = AC$.

**Postulate 1-2-2** Segment Addition Postulate

If $B$ is between $A$ and $C$, then $AB + BC = AC$. 

14  Chapter 1 Foundations for Geometry
**Example 3**

**Using the Segment Addition Postulate**

**A**

$B$ is between $A$ and $C$, $AC = 14$, and $BC = 11.4$. Find $AB$.

\[ \begin{align*}
AC &= AB + BC & \text{Seg. Add. Post.} \\
14 &= AB + 11.4 & \text{Substitute 14 for $AC$ and 11.4 for $BC$.} \\
-11.4 &= -11.4 & \text{Subtract 11.4 from both sides.} \\
2.6 &= AB & \text{Simplify.}
\end{align*} \]

**B**

$S$ is between $R$ and $T$. Find $RT$.

\[ \begin{align*}
RT &= RS + ST & \text{Seg. Add. Post.} \\
4x &= (2x + 7) + 28 & \text{Substitute the given values.} \\
4x &= 2x + 35 & \text{Simplify.} \\
-2x &= -2x & \text{Subtract } 2x \text{ from both sides.} \\
2x &= 35 & \text{Simplify.} \\
\frac{2x}{2} &= \frac{35}{2} & \text{Divide both sides by 2.} \\
x &= 17.5, \text{ or } 17.5 & \text{Simplify.} \\
RT &= 4x \hspace{2cm} \text{Substitute } 17.5 \text{ for } x.
\end{align*} \]

**Check It Out!**

3a. $Y$ is between $X$ and $Z$, $XZ = 3$, and $XY = 1 \frac{1}{3}$. Find $YZ$.

3b. $E$ is between $D$ and $F$. Find $DF$.

The midpoint $M$ of $\overline{AB}$ is the point that **bisects**, or divides, the segment into two congruent segments. If $M$ is the midpoint of $\overline{AB}$, then $AM = MB$. So if $AB = 6$, then $AM = 3$ and $MB = 3$.

**Example 4**

**Recreation Application**

The map shows the route for a race.

You are 365 m from drink station $R$ and 2 km from drink station $S$.

The first-aid station is located at the midpoint of the two drink stations. How far are you from the first-aid station?

Let your current location be $X$ and the location of the first-aid station be $Y$.

\[ \begin{align*}
XR + RS &= XS & \text{Seg. Add. Post.} \\
365 + RS &= 2000 & \text{Substitute 365 for $XR$ and 2000 for $XS$.} \\
-365 &= -365 & \text{Subtract 365 from both sides.} \\
RS &= 1635 & \text{Simplify.} \\
RY &= 817.5 & \text{$Y$ is the mdpt. of $RS$, so $RY = \frac{1}{2}RS$.} \\
XY &= XR + RY \hspace{2cm} \text{Substitute 365 for $XR$ and 817.5 for $RY$.} \\
&= 365 + 817.5 = 1182.5 \text{ m}
\end{align*} \]

You are 1182.5 m from the first-aid station.

**Check It Out!**

4. What is the distance to a drink station located at the midpoint between your current location and the first-aid station?
A **segment bisector** is any ray, segment, or line that intersects a segment at its midpoint. It divides the segment into two equal parts at its midpoint.

**Construction: Segment Bisector**

1. Draw $XY$ on a sheet of paper.
2. Fold the paper so that $Y$ is on top of $X$.
3. Unfold the paper. The line represented by the crease bisects $XY$. Label the midpoint $M$.

**Using Midpoints to Find Lengths**

**Example 5**

$B$ is the midpoint of $\overline{AC}$, $AB = 5x$, and $BC = 3x + 4$. Find $AB$, $BC$, and $AC$.

**Step 1** Solve for $x$.

\[
\begin{align*}
AB &= BC \\
5x &= 3x + 4 \\
-3x &= 3x \\
2x &= 4 \\
\frac{2x}{2} &= \frac{4}{2} \\
x &= 2
\end{align*}
\]

**Step 2** Find $AB$, $BC$, and $AC$.

\[
\begin{align*}
AB &= 5x \\
BC &= 3x + 4 \\
AC &= AB + BC \\
&= 5(2) + 10 \\
&= 10 + 10 = 20
\end{align*}
\]

**Check It Out!**

5. $S$ is the midpoint of $RT$, $RS = -2x$, and $ST = -3x - 2$. Find $RS$, $ST$, and $RT$.

**Think and Discuss**

1. Suppose $R$ is the midpoint of $ST$. Explain how $SR$ and $ST$ are related.
2. **Get Organized** Copy and complete the graphic organizer. Make a sketch and write an equation to describe each relationship.
**GUIDED PRACTICE**

**Vocabulary**  Apply the vocabulary from this lesson to answer each question.

1. Line \( \ell \) bisects \( \overline{XY} \) at \( M \) and divides \( \overline{XY} \) into two equal parts. Name a pair of congruent segments.

2. \( \text{?} \) is the amount of space between two points on a line. It is always expressed as a nonnegative number. (distance or midpoint)

---

**See Example 1**

Find each length.

3. \( \overline{AB} \)

4. \( \overline{BC} \)

---

**See Example 2**

5. Sketch, draw, and construct a segment congruent to \( \overline{RS} \).

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**See Example 3**

6. \( B \) is between \( A \) and \( C \), \( AC = 15.8 \), and \( AB = 9.9 \). Find \( BC \).

7. Find \( MP \).

---

**See Example 4**

8. **Travel** If a picnic area is located at the midpoint between Sacramento and Oakland, find the distance to the picnic area from the road sign.

---

**See Example 5**

9. **Multi-Step** \( K \) is the midpoint of \( \overline{JL} \), \( JL = 4x - 2 \), and \( JK = 7 \). Find \( x \), \( KL \), and \( JL \).

10. \( E \) bisects \( \overline{DF} \), \( DE = 2y \), and \( EF = 8y - 3 \). Find \( DE \), \( EF \), and \( DF \).

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**PRACTICE AND PROBLEM SOLVING**

Find each length.

11. \( \overline{DB} \)

12. \( \overline{CD} \)

13. Sketch, draw, and construct a segment twice the length of \( \overline{AB} \).

14. \( D \) is between \( C \) and \( E \), \( CE = 17.1 \), and \( DE = 8 \). Find \( CD \).

15. Find \( MN \).

16. **Sports** During a football game, a quarterback standing at the 9-yard line passes the ball to a receiver at the 24-yard line. The receiver then runs with the ball halfway to the 50-yard line. How many total yards (passing plus running) did the team gain on the play?

17. **Multi-Step** \( E \) is the midpoint of \( \overline{DF} \), \( DE = 2x + 4 \), and \( EF = 3x - 1 \). Find \( DE \), \( EF \), and \( DF \).

18. \( Q \) bisects \( \overline{PR} \), \( PQ = 3y \), and \( PR = 42 \). Find \( y \) and \( QR \).
19. This problem will prepare you for the Multi-Step Test Prep on page 34. Archaeologists at Valley Forge were eager to find what remained of the winter camp that soldiers led by George Washington called home for several months. The diagram represents one of the restored log cabins.

a. How is \( C \) related to \( \overline{AE} \)?

b. If \( AC = 7 \text{ ft} \), \( EF = 2(AC) + 2 \), and \( AB = 2(EF) - 16 \), what are \( AB \) and \( EF \)?

Use the diagram for Exercises 20–23.

20. \( GD = 4 \frac{2}{3} \). Find \( GH \).

21. \( \overline{CD} \equiv \overline{DF} \), \( E \) bisects \( \overline{DF} \), and \( CD = 14.2 \). Find \( EF \).

22. \( GH = 4x - 1 \), and \( DH = 8 \). Find \( x \).

23. \( \overline{GH} \) bisects \( \overline{CF} \), \( CF = 2y - 2 \), and \( CD = 3y - 11 \). Find \( CD \).

Tell whether each statement is sometimes, always, or never true. Support each of your answers with a sketch.

24. Two segments that have the same length must be congruent.

25. If \( M \) is between \( A \) and \( B \), then \( M \) bisects \( \overline{AB} \).

26. If \( Y \) is between \( X \) and \( Z \), then \( X, Y, \) and \( Z \) are collinear.

27. \( \square \) ERROR ANALYSIS \( \square \) Below are two statements about the midpoint of \( \overline{AB} \). Which is incorrect? Explain the error.

28. Carpentry A carpenter has a wooden dowel that is 72 cm long. She wants to cut it into two pieces so that one piece is 5 times as long as the other. What are the lengths of the two pieces?

29. The coordinate of \( M \) is 2.5, and \( MN = 4 \). What are the possible coordinates for \( N \)?

30. Draw three collinear points where \( E \) is between \( D \) and \( F \). Then write an equation using these points and the Segment Addition Postulate.

Suppose \( S \) is between \( R \) and \( T \). Use the Segment Addition Postulate to solve for each variable.

31. \( RS = 7y - 4 \)
   \( ST = y + 5 \)
   \( RT = 28 \)

32. \( RS = 3x + 1 \)
   \( ST = \frac{1}{2}x + 3 \)
   \( RT = 18 \)

33. \( RS = 2z + 6 \)
   \( ST = 4z - 3 \)
   \( RT = 5z + 12 \)

34. Write About It In the diagram, \( B \) is not between \( A \) and \( C \). Explain.

35. Construction Use a compass and straightedge to construct a segment whose length is \( AB + CD \).
36. Q is between P and R. S is between Q and R, and R is between Q and T. PT = 34, QR = 8, and PQ = SQ = SR. What is the length of RT?
   \[\text{A} \quad 9 \quad \text{B} \quad 10 \quad \text{C} \quad 18 \quad \text{D} \quad 22\]

37. C is the midpoint of AD. B is the midpoint of AC. BC = 12. What is the length of AD?
   \[\text{F} \quad 12 \quad \text{G} \quad 24 \quad \text{H} \quad 36 \quad \text{J} \quad 48\]

38. Which expression correctly states that XY is congruent to VW?
   \[\text{A} \quad XY \equiv VW \quad \text{B} \quad XY \cong VW \quad \text{C} \quad XY = VW \quad \text{D} \quad XY = VW\]

39. A, B, C, D, and E are collinear points. AE = 34, BD = 16, and AB = BC = CD. What is the length of CE?
   \[\text{F} \quad 10 \quad \text{G} \quad 16 \quad \text{H} \quad 18 \quad \text{I} \quad 24\]

**CHALLENGE AND EXTEND**

40. HJ is twice JK. J is between H and K. If HJ = 4x and HK = 78, find JK.

41. A, D, N, and X are collinear points. D is between N and A. NA + AX = NX. Draw a diagram that represents this information.

**Sports** Use the following information for Exercises 42 and 43.

The table shows regulation distances between hurdles in women's and men's races. In both the women's and men's events, the race consists of a straight track with 10 equally spaced hurdles.

<table>
<thead>
<tr>
<th>Event</th>
<th>Distance of Race</th>
<th>Distance from Start to First Hurdle</th>
<th>Distance Between Hurdles</th>
<th>Distance from Last Hurdle to Finish</th>
</tr>
</thead>
<tbody>
<tr>
<td>Women's</td>
<td>100 m</td>
<td>13.00 m</td>
<td>8.50 m</td>
<td></td>
</tr>
<tr>
<td>Men's</td>
<td>110 m</td>
<td>13.72 m</td>
<td>9.14 m</td>
<td></td>
</tr>
</tbody>
</table>

42. Find the distance from the last hurdle to the finish line for the women’s race.

43. Find the distance from the last hurdle to the finish line for the men’s race.

44. **Critical Thinking** Given that J, K, and L are collinear and that K is between J and L, is it possible that JK = JL? If so, draw an example. If not, explain.

**SPIRAL REVIEW**

Evaluate each expression. *(Previous course)*

45. \[|20 - 8|\]

46. \[|-9 + 23|\]

47. \[|-4 - 27|\]

Simplify each expression. *(Previous course)*

48. \[8a - 3(4 + a) - 10\]

49. \[x + 2(5 - 2x) - (4 + 5x)\]

Use the figure to name each of the following. *(Lesson 1-1)*

50. two lines that contain B

51. two segments containing D

52. three collinear points

53. a ray with endpoint C
Objectives
Name and classify angles.
Measure and construct angles and angle bisectors.

Vocabulary
angle
vertex
interior of an angle
exterior of an angle
measure
degree
acute angle
right angle
obtuse angle
straight angle
congruent angles
angle bisector

Who uses this?
Surveyors use angles to help them measure and map the earth's surface. (See Exercise 27.)

A transit is a tool for measuring angles. It consists of a telescope that swivels horizontally and vertically. Using a transit, a surveyor can measure the angle formed by his or her location and two distant points.

An angle is a figure formed by two rays, or sides, with a common endpoint called the vertex (plural: vertices). You can name an angle several ways: by its vertex, by a point on each ray and the vertex, or by a number.

The set of all points between the sides of the angle is the interior of an angle. The exterior of an angle is the set of all points outside the angle.

Angle Name
∠R, ∠SRT, ∠TRS, or ∠1

You cannot name an angle just by its vertex if the point is the vertex of more than one angle. In this case, you must use all three points to name the angle, and the middle point is always the vertex.

Example 1
Naming Angles
A surveyor recorded the angles formed by a transit (point T) and three distant points, Q, R, and S. Name three of the angles.

∠QTR, ∠QTS, and ∠RTS

1. Write the different ways you can name the angles in the diagram.

The measure of an angle is usually given in degrees. Since there are 360° in a circle, one degree is \(\frac{1}{360}\) of a circle. When you use a protractor to measure angles, you are applying the following postulate.

Postulate 1-3-1 Protractor Postulate
Given AB and a point O on AB, all rays that can be drawn from O can be put into a one-to-one correspondence with the real numbers from 0 to 180.
You can use the Protractor Postulate to help you classify angles by their measure. The measure of an angle is the absolute value of the difference of the real numbers that the rays correspond with on a protractor. If $\overrightarrow{OC}$ corresponds with $c$ and $\overrightarrow{OD}$ corresponds with $d$, $m\angle DOC = |d - c|$ or $|c - d|$.

Types of Angles

- **Acute Angle**: Measures greater than 0° and less than 90°
- **Right Angle**: Measures 90°
- **Obtuse Angle**: Measures greater than 90° and less than 180°
- **Straight Angle**: Formed by two opposite rays and measures 180°

**EXAMPLE 2** Measuring and Classifying Angles

Find the measure of each angle. Then classify each as acute, right, or obtuse.

**A** \(\angle AOD\)

$m\angle AOD = 165^\circ$

\(\angle AOD\) is obtuse.

**B** \(\angle COD\)

$m\angle COD = |165 - 75| = 90^\circ$

\(\angle COD\) is a right angle.

Use the diagram to find the measure of each angle. Then classify each as acute, right, or obtuse.

2a. \(\angle BOA\)  
2b. \(\angle DOB\)  
2c. \(\angle EOC\)
**Congruent angles** are angles that have the same measure. In the diagram, \( \angle ABC = \angle DEF \), so you can write \( \angle ABC \cong \angle DEF \). This is read as “angle \( \triangle ABC \) is congruent to angle \( \triangle DEF \).” **Arc marks** are used to show that the two angles are congruent.

**Construction: Congruent Angle**

**Example:**

Construct an angle congruent to \( \angle A \).

1. Use a straightedge to draw a ray with endpoint \( D \).
2. Place the compass point at \( A \) and draw an arc that intersects both sides of \( \angle A \). Label the intersection points \( B \) and \( C \).
3. Using the same compass setting, place the compass point at \( D \) and draw an arc that intersects the ray. Label the intersection \( E \).
4. Place the compass point at \( B \) and open it to the distance \( BC \). Place the point of the compass at \( E \) and draw an arc. Label its intersection with the first arc \( F \).
5. Use a straightedge to draw \( \overline{DF} \).

\( \angle D \cong \angle A \)

The Angle Addition Postulate is very similar to the Segment Addition Postulate that you learned in the previous lesson.

**Postulate 1-3-2 Angle Addition Postulate**

If \( S \) is in the interior of \( \angle PQR \), then

\[ m\angle PQS + m\angle SQR = m\angle PQR. \]

(\( \angle \) Add. Post.)

**Example:**

Using the Angle Addition Postulate

\( m\angle ABD = 37^\circ \) and \( m\angle ABC = 84^\circ \). Find \( m\angle BDC \).

\[
m\angle ABC = m\angle ABD + m\angle BDC \quad \angle \text{Add. Post.}
\]

\[
84^\circ = 37^\circ + m\angle BDC
\]

\[
-37 -37
\]

\[
47^\circ = m\angle BDC
\]

\( m\angle XWZ = 121^\circ \) and \( m\angle XWY = 59^\circ \). Find \( m\angle YWZ \).
An **angle bisector** is a ray that divides an angle into two congruent angles. \( \overrightarrow{JK} \) bisects \( \angle LJM \); thus \( \angle LJK \cong \angle KJM \).

### Construction Angle Bisector

**Construct the bisector of \( \angle A \).**

1. Place the point of the compass at \( A \) and draw an arc. Label its points of intersection with \( \angle A \) as \( B \) and \( C \).
2. Without changing the compass setting, draw intersecting arcs from \( B \) and \( C \). Label the intersection of the arcs as \( D \).
3. Use a straightedge to draw \( \overrightarrow{AD} \). \( \overrightarrow{AD} \) bisects \( \angle A \).

### Example 4

**Finding the Measure of an Angle**

\( \overrightarrow{BD} \) bisects \( \angle ABC \), \( \text{m}\angle ABD = (6x + 3)° \), and \( \text{m}\angle DBC = (8x - 7)° \). Find \( \text{m}\angle ABD \).

**Step 1** Find \( x \).

\[
\begin{align*}
\text{m}\angle ABD &= \text{m}\angle DBC \\
(6x + 3)^° &= (8x - 7)^° \\
\underline{+ 7} & \quad \underline{+ 7} \\
6x + 10 &= 8x \\
-6x &= -6x \\
10 &= 2x \\
\frac{10}{2} &= \frac{2x}{2} \\
5 &= x
\end{align*}
\]

**Step 2** Find \( \text{m}\angle ABD \).

\[
\begin{align*}
\text{m}\angle ABD &= 6x + 3 \\
&= 6(5) + 3 \\
&= 33°
\end{align*}
\]

Find the measure of each angle.

4a. \( \overrightarrow{QS} \) bisects \( \angle PQR \), \( \text{m}\angle PQS = (5y - 1)^° \), and \( \text{m}\angle PQR = (8y + 12)^° \). Find \( \text{m}\angle PQS \).

4b. \( \overrightarrow{JK} \) bisects \( \angle LJM \), \( \text{m}\angle LJK = (-10x + 3)^° \), and \( \text{m}\angle KJM = (-x + 21)^° \). Find \( \text{m}\angle LJM \).
1. Explain why any two right angles are congruent.

2. **BD** bisects \( \angle ABC \). How are \( \angle ABC \), \( \angle ABD \), and \( \angle DBC \) related?

3. **GET ORGANIZED** Copy and complete the graphic organizer. In the cells sketch, measure, and name an example of each angle type.

<table>
<thead>
<tr>
<th>Diagram</th>
<th>Measure</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acute Angle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Right Angle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obtuse Angle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Straight Angle</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**THINK AND DISCUSS**

1. Explain why any two right angles are congruent.

2. **BD** bisects \( \angle ABC \). How are \( \angle ABC \), \( \angle ABD \), and \( \angle DBC \) related?

3. **GET ORGANIZED** Copy and complete the graphic organizer. In the cells sketch, measure, and name an example of each angle type.

1. \( \angle A \) is an acute angle. \( \angle O \) is an obtuse angle. \( \angle R \) is a right angle. Put \( \angle A \), \( \angle O \), and \( \angle R \) in order from least to greatest by measure.

2. Which point is the vertex of \( \angle BCD \)? Which rays form the sides of \( \angle BCD \)?

3. **Music** Musicians use a metronome to keep time as they play. The metronome's needle swings back and forth in a fixed amount of time. Name all of the angles in the diagram.

4. Use the protractor to find the measure of each angle. Then classify each as acute, right, or obtuse.
   4. \( \angle VXW \)
   5. \( \angle TXW \)
   6. \( \angle RXU \)

5. **L** is in the interior of \( \angle JKM \). Find each of the following.
   7. \( m\angle JKM \) if \( m\angle JKL = 42^\circ \) and \( m\angle LKM = 28^\circ \)
   8. \( m\angle LKM \) if \( m\angle JKL = 56.4^\circ \) and \( m\angle JKM = 82.5^\circ \)

6. **Multi-Step** \( \overrightarrow{BD} \) bisects \( \angle ABC \). Find each of the following.
   9. \( m\angle ABD \) if \( m\angle ABD = (6x + 4)^\circ \) and \( m\angle DBC = (8x - 4)^\circ \)
   10. \( m\angle ABC \) if \( m\angle ABD = (5y - 3)^\circ \) and \( m\angle DBC = (3y + 15)^\circ \)
11. **Physics** Pen-dulum clocks have been used since 1656 to keep time. The pendulum swings back and forth once or twice per second. Name all of the angles in the diagram.

Use the protractor to find the measure of each angle. Then classify each as acute, right, or obtuse.

12. \( \angle CGE \)
13. \( \angle BGD \)
14. \( \angle AGB \)

T is in the interior of \( \angle RSU \). Find each of the following.

15. \( m\angle RSU \) if \( m\angle RST = 38° \) and \( m\angle TSU = 28.6° \)

16. \( m\angle RST \) if \( m\angle TSU = 46.7° \) and \( m\angle RSU = 83.5° \)

**Multi-Step** \( \overline{SP} \) bisects \( \angle RST \). Find each of the following.

17. \( m\angle RST \) if \( m\angle RSP = (3x - 2)° \) and \( m\angle PST = (9x - 26)° \)

18. \( m\angle RSP \) if \( m\angle RST = \frac{3}{2}y° \) and \( m\angle PST = (y + 5)° \)

**Estimation** Use the following information for Exercises 19–22.

Assume the corner of a sheet of paper is a right angle. Use the corner to estimate the measure and classify each angle in the diagram.

19. \( \angle BOA \)
20. \( \angle COA \)
21. \( \angle EOD \)
22. \( \angle EOB \)

Use a protractor to draw an angle with each of the following measures.

23. 33°
24. 142°
25. 90°
26. 168°

27. **Surveying** A surveyor at point S discovers that the angle between peaks A and B is 3 times as large as the angle between peaks B and C. The surveyor knows that \( \angle ASC \) is a right angle. Find \( m\angle ASB \) and \( m\angle BSC \).

28. **Math History** As far back as the 5th century B.C., mathematicians have been fascinated by the problem of trisecting an angle. It is possible to construct an angle with \( \frac{1}{4} \) the measure of a given angle. Explain how to do this.

Find the value of \( x \).

29. \( m\angle AOC = 7x - 2, m\angle DOC = 2x + 8, m\angle EOD = 27 \)
30. \( m\angle AOB = 4x - 2, m\angle BOC = 5x + 10, m\angle COD = 3x - 8 \)
31. \( m\angle AOB = 6x + 5, m\angle BOC = 4x - 2, m\angle AOC = 8x + 21 \)
32. **Multi-Step** Q is in the interior of right \( \angle PRS \). If \( m\angle PRQ \) is 4 times as large as \( m\angle QRS \), what is \( m\angle PRQ \)?
33. This problem will prepare you for the Multi-Step Test Prep on page 34. An archaeologist standing at O looks for clues on where to dig for artifacts.
   a. What value of x will make the angle between the pottery and the arrowhead measure 57°?
   b. What value of x makes \( \angle LOJ \cong \angle JOK \)?
   c. What values of x make \( \angle JOK \) an acute angle?

Data Analysis Use the circle graph for Exercises 34–36.

34. Find \( m\angle AOB \), \( m\angle BOC \), \( m\angle COD \), and \( m\angle DOA \). Classify each angle as acute, right, or obtuse.

35. What if…? Next year, the music store will use some of the shelves currently holding jazz music to double the space for rap. What will \( m\angle COD \) and \( m\angle BOC \) be next year?

36. Suppose a fifth type of music, salsa, is added. If the space is divided equally among the five types, what will be the angle measure for each type of music in the circle graph?

37. Critical Thinking Can an obtuse angle be congruent to an acute angle? Why or why not?

38. The measure of an obtuse angle is \( (5x + 45)^\circ \). What is the largest value for \( x \)?

39. Write About It \( FH \) bisects \( \angle EFG \). Use the Angle Addition Postulate to explain why \( m\angle EFH = \frac{1}{2}m\angle EFG \).

40. Multi-Step Use a protractor to draw a 70° angle. Then use a compass and straightedge to bisect the angle. What do you think will be the measure of each angle formed? Use a protractor to support your answer.

41. \( m\angle UOW = 50^\circ \), and \( OV \) bisects \( \angle UOW \).
   What is \( m\angle VOY \)?
   \[ \begin{array}{ll}
   A & 25^\circ \\
   B & 65^\circ \\
   C & 130^\circ \\
   D & 155^\circ \\
   \end{array} \]

42. What is \( m\angle UOX \)?
   \[ \begin{array}{ll}
   A & 50^\circ \\
   B & 115^\circ \\
   C & 140^\circ \\
   D & 165^\circ \\
   \end{array} \]

43. \( BD \) bisects \( \angle ABC \), \( m\angle ABC = (4x + 5)^\circ \), and \( m\angle ABD = (3x - 1)^\circ \).
   What is the value of \( x \)?
   \[ \begin{array}{ll}
   A & 2.2 \\
   B & 3 \\
   C & 3.5 \\
   D & 7 \\
   \end{array} \]

44. If an angle is bisected and then 30° is added to the measure of the bisected angle, the result is the measure of a right angle. What is the measure of the original angle?
   \[ \begin{array}{ll}
   A & 30^\circ \\
   B & 60^\circ \\
   C & 75^\circ \\
   D & 120^\circ \\
   \end{array} \]

45. Short Response If an obtuse angle is bisected, are the resulting angles acute or obtuse? Explain.
**CHALLENGE AND EXTEND**

46. Find the measure of the angle formed by the hands of a clock when it is 7:00.

47. \(QS\) bisects \( \angle PQR \), \( m\angle PQR = (x^2)^\circ \), and \( m\angle PQS = (2x + 6)^\circ \). Find all the possible measures for \( \angle PQR \).

48. For more precise measurements, a degree can be divided into 60 minutes, and each minute can be divided into 60 seconds. An angle measure of 42 degrees, 30 minutes, and 10 seconds is written as 42°30′10″. Subtract this angle measure from the measure 81°24′15″.

49. If 1 degree equals 60 minutes and 1 minute equals 60 seconds, how many seconds are in 2.25 degrees?

50. \( \angle ABC \cong \angle DBC \). \( m\angle ABC = \left( \frac{3x}{2} + 4 \right)^\circ \) and \( m\angle DBC = \left( 2x - 27\frac{1}{4} \right)^\circ \). Is \( \angle ABD \) a straight angle? Explain.

**SPIRAL REVIEW**

51. What number is 64% of 35?

52. What percent of 280 is 33.6? (Previous course)

53. Sketch a figure that shows each of the following. (Lesson 1-1)
   a. a line that contains \( \overline{AB} \) and \( \overline{CB} \)
   b. two different lines that intersect \( \overline{MN} \)
   c. a plane and a ray that intersect only at \( Q \)

54. Find the length of each segment. (Lesson 1-2)
   a. \( \overline{JK} \)
   b. \( \overline{KL} \)
   c. \( \overline{JL} \)

---

**Using Technology** Segment and Angle Bisectors

1. Construct the bisector of \( \overline{MN} \).
   a. Draw \( \overline{MN} \) and construct the midpoint \( B \).
   b. Construct a point \( A \) not on the segment.
   c. Construct bisector \( \overline{AB} \) and measure \( MB \) and \( NB \).
   d. Drag \( M \) and \( N \) and observe \( MB \) and \( NB \).

2. Construct the bisector of \( \angle BAC \).
   a. Draw \( \angle BAC \).
   b. Construct the angle bisector \( \overline{AD} \) and measure \( \angle DAC \) and \( \angle DAB \).
   c. Drag the angle and observe \( m \angle DAB \) and \( m \angle DAC \).
Who uses this?

Scientists use properties of angle pairs to design fiber-optic cables. (See Example 4.)

A fiber-optic cable is a strand of glass as thin as a human hair. Data can be transmitted over long distances by bouncing light off the inner walls of the cable.

Many pairs of angles have special relationships. Some relationships are because of the measurements of the angles in the pair. Other relationships are because of the positions of the angles in the pair.

Objectives

Identify adjacent, vertical, complementary, and supplementary angles.

Find measures of pairs of angles.

Vocabulary

adjacent angles
linear pair
complementary angles
supplementary angles
vertical angles

Pairs of Angles

Adjacent angles are two angles in the same plane with a common vertex and a common side, but no common interior points. \( \angle 1 \) and \( \angle 2 \) are adjacent angles.

A linear pair of angles is a pair of adjacent angles whose noncommon sides are opposite rays. \( \angle 3 \) and \( \angle 4 \) form a linear pair.

Example 1

Identifying Angle Pairs

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

A. \( \angle 1 \) and \( \angle 2 \)

\( \angle 1 \) and \( \angle 2 \) have a common vertex, \( B \), a common side, \( \overline{BC} \), and no common interior points. Therefore \( \angle 1 \) and \( \angle 2 \) are only adjacent angles.

B. \( \angle 2 \) and \( \angle 4 \)

\( \angle 2 \) and \( \angle 4 \) share \( \overline{BC} \) but do not have a common vertex, so \( \angle 2 \) and \( \angle 4 \) are not adjacent angles.

C. \( \angle 1 \) and \( \angle 3 \)

\( \angle 1 \) and \( \angle 3 \) are adjacent angles. Their noncommon sides, \( \overline{BC} \) and \( \overline{BA} \), are opposite rays, so \( \angle 1 \) and \( \angle 3 \) also form a linear pair.

Check It Out!

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

1a. \( \angle 5 \) and \( \angle 6 \)

1b. \( \angle 7 \) and \( \angle SPU \)

1c. \( \angle 7 \) and \( \angle 8 \)
Complementary and Supplementary Angles

Complementary angles are two angles whose measures have a sum of 90°.
\[ \angle A \text{ and } \angle B \text{ are complementary.} \]

Supplementary angles are two angles whose measures have a sum of 180°.
\[ \angle A \text{ and } \angle C \text{ are supplementary.} \]

You can find the complement of an angle that measures \( x \)° by subtracting its measure from 90°, or \((90 - x)°\). You can find the supplement of an angle that measures \( x \)° by subtracting its measure from 180°, or \((180 - x)°\).

**Example 2** Finding the Measures of Complements and Supplements

Find the measure of each of the following.

A. complement of \( \angle M \)
\[(90 - x)°\]
\[90° - 26.8° = 63.2°\]

B. supplement of \( \angle N \)
\[(180 - x)°\]
\[180° - (2y + 20)° = 180° - 2y - 20° = (160 - 2y)°\]

Find the measure of each of the following.

2a. complement of \( \angle E \)
2b. supplement of \( \angle F \)

**Example 3** Using Complements and Supplements to Solve Problems

An angle measures 3 degrees less than twice the measure of its complement. Find the measure of its complement.

Step 1 Let \( m\angle A = x° \). Then \( \angle B \), its complement, measures \((90 - x)°\).

Step 2 Write and solve an equation.
\[ m\angle A = 2m\angle B - 3 \]
\[ x = 2(90 - x) - 3 \]
\[ x = 180 - 2x - 3 \]
\[ x = 177 - 2x \]
\[ + 2x + 2x \]
\[ 3x = 177 \]
\[ 3x = \frac{177}{3} \]
\[ x = 59 \]

The measure of the complement, \( \angle B \), is \((90 - 59)° = 31°\).

3. An angle’s measure is 12° more than \( \frac{1}{2} \) the measure of its supplement. Find the measure of the angle.
Problem-Solving Application

Light passing through a fiber optic cable reflects off the walls in such a way that \( \angle 1 \cong \angle 2 \). \( \angle 3 \) and \( \angle 4 \) are complementary, and \( \angle 2 \) and \( \angle 4 \) are complementary.

If \( m\angle 1 = 38^\circ \), find \( m\angle 2 \), \( m\angle 3 \), and \( m\angle 4 \).

1. Understand the Problem

The answers are the measures of \( \angle 2 \), \( \angle 3 \), and \( \angle 4 \).

List the important information:
- \( \angle 1 \cong \angle 2 \)
- \( \angle 1 \) and \( \angle 3 \) are complementary, and \( \angle 2 \) and \( \angle 4 \) are complementary.
- \( m\angle 1 = 38^\circ \)

2. Make a Plan

If \( \angle 1 \cong \angle 2 \), then \( m\angle 1 = m\angle 2 \).
If \( \angle 3 \) and \( \angle 1 \) are complementary, then \( m\angle 3 = (90 - 38)^\circ \).
If \( \angle 4 \) and \( \angle 2 \) are complementary, then \( m\angle 4 = (90 - 38)^\circ \).

3. Solve

By the Transitive Property of Equality, if \( m\angle 1 = 38^\circ \) and \( m\angle 1 = m\angle 2 \), then \( m\angle 2 = 38^\circ \). Since \( \angle 3 \) and \( \angle 1 \) are complementary, \( m\angle 3 = 52^\circ \). Similarly, since \( \angle 2 \) and \( \angle 4 \) are complementary, \( m\angle 4 = 52^\circ \).

4. Look Back

The answer makes sense because \( 38^\circ + 52^\circ = 90^\circ \), so \( \angle 1 \) and \( \angle 3 \) are complementary, and \( \angle 2 \) and \( \angle 4 \) are complementary. Thus \( m\angle 2 = 38^\circ \), \( m\angle 3 = 52^\circ \), and \( m\angle 4 = 52^\circ \).

4. What if...? Suppose \( m\angle 3 = 27.6^\circ \). Find \( m\angle 1 \), \( m\angle 2 \), and \( m\angle 4 \).

Another angle pair relationship exists between two angles whose sides form two pairs of opposite rays. Vertical angles are two nonadjacent angles formed by two intersecting lines. \( \angle 1 \) and \( \angle 3 \) are vertical angles, as are \( \angle 2 \) and \( \angle 4 \).

Example 5

Identifying Vertical Angles

Name one pair of vertical angles. Do they appear to have the same measure? Check by measuring with a protractor.

\( \angle EDF \) and \( \angle GDH \) are vertical angles and appear to have the same measure.

Check \( m\angle EDF \approx m\angle GDH \approx 135^\circ \).

5. Name another pair of vertical angles. Do they appear to have the same measure? Check by measuring with a protractor.
THINK AND DISCUSS
1. Explain why any two right angles are supplementary.
2. Is it possible for a pair of vertical angles to also be adjacent? Explain.
3. GET ORGANIZED Copy and complete the graphic organizer below. In each box, draw a diagram and write a definition of the given angle pair.

Exercises

GUIDED PRACTICE
Vocabulary Apply the vocabulary from this lesson to answer each question.
1. An angle measures $x^\circ$. What is the measure of its complement? What is the measure of its supplement?

2. $\angle ABC$ and $\angle CBD$ are adjacent angles. Which side do the angles have in common?

3. $\angle 1$ and $\angle 2$
4. $\angle 1$ and $\angle 3$
5. $\angle 2$ and $\angle 4$
6. $\angle 2$ and $\angle 3$

7. Find the measure of each of the following.
8. Find the measure of each of the following.
9. Find the measure of each of the following.
10. Find the measure of each of the following.
11. Multi-Step An angle's measure is 6 degrees more than 3 times the measure of its complement. Find the measure of the angle.

12. Landscaping A sprinkler swings back and forth between $A$ and $B$ in such a way that $\angle 1 \equiv \angle 2$. $\angle 1$ and $\angle 3$ are complementary, and $\angle 2$ and $\angle 4$ are complementary. If $m\angle 1 = 47.5^\circ$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.

13. Name each pair of vertical angles.
**PRACTICE AND PROBLEM SOLVING**

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

14. \( \angle 1 \) and \( \angle 4 \)  
15. \( \angle 2 \) and \( \angle 3 \)  
16. \( \angle 3 \) and \( \angle 4 \)  
17. \( \angle 3 \) and \( \angle 1 \)

Given \( m\angle A = 56.4^\circ \) and \( m\angle B = (2x - 4)^\circ \), find the measure of each of the following.

18. supplement of \( \angle A \)  
19. complement of \( \angle A \)  
20. supplement of \( \angle B \)  
21. complement of \( \angle B \)

**Multi-Step** An angle’s measure is 3 times the measure of its complement. Find the measure of the angle and the measure of its complement.

23. **Art** In the stained glass pattern, \( \angle 1 \cong \angle 2 \). \( \angle 1 \) and \( \angle 3 \) are complementary, and \( \angle 2 \) and \( \angle 4 \) are complementary. If \( m\angle 1 = 22.3^\circ \), find \( m\angle 2 \), \( m\angle 3 \), and \( m\angle 4 \).

24. Name the pairs of vertical angles.

25. **Probability** The angle measures 30°, 60°, 120°, and 150° are written on slips of paper. You choose two slips of paper at random. What is the probability that the angle measures are supplementary?

**Multi-Step** \( \angle ABD \) and \( \angle BDE \) are supplementary. Find the measures of both angles.

26. \( m\angle ABD = 5x^\circ \), \( m\angle BDE = (17x - 18)^\circ \)  
27. \( m\angle ABD = (3x + 12)^\circ \), \( m\angle BDE = (7x - 32)^\circ \)  
28. \( m\angle ABD = (12x - 12)^\circ \), \( m\angle BDE = (3x + 48)^\circ \)

**Multi-Step** \( \angle ABD \) and \( \angle BDC \) are complementary. Find the measures of both angles.

29. \( m\angle ABD = (5y + 1)^\circ \), \( m\angle BDC = (3y - 7)^\circ \)  
30. \( m\angle ABD = (4y + 5)^\circ \), \( m\angle BDC = (4y + 8)^\circ \)  
31. \( m\angle ABD = (y - 30)^\circ \), \( m\angle BDC = 2y^\circ \)

32. **Critical Thinking** Explain why an angle that is supplementary to an acute angle must be an obtuse angle.

33. This problem will prepare you for the Multi-Step Test Prep on page 34. \( H \) is in the interior of \( \angle JAK \). \( m\angle JAH = (3x - 8)^\circ \), and \( m\angle KAH = (x + 2)^\circ \). Draw a picture of each relationship. Then find the measure of each angle.
   a. \( \angle JAH \) and \( \angle KAH \) are complementary angles.
   b. \( \angle JAH \) and \( \angle KAH \) form a linear pair.
   c. \( \angle JAH \) and \( \angle KAH \) are congruent angles.
Determine whether each statement is true or false. If false, explain why.

34. If an angle is acute, then its complement must be greater than its supplement.

35. A pair of vertical angles may also form a linear pair.

36. If two angles are supplementary and congruent, the measure of each angle is 90°.

37. If a ray divides an angle into two complementary angles, then the original angle is a right angle.

38. Write About It Describe a situation in which two angles are both congruent and complementary. Explain.

39. What is the value of \( x \) in the diagram?

40. The ratio of the measures of two complementary angles is 1:2. What is the measure of the larger angle? (Hint: Let \( x \) and \( 2x \) represent the angle measures.)

41. \( m\angle A = 3y \), and \( m\angle B = 2m\angle A \). Which value of \( y \) makes \( \angle A \) supplementary to \( \angle B \)?

42. The measures of two supplementary angles are in the ratio 7:5. Which value is the measure of the smaller angle? (Hint: Let \( 7x \) and \( 5x \) represent the angle measures.)

**CHALLENGE AND EXTEND**

43. How many pairs of vertical angles are in the diagram?

44. The supplement of an angle is 4 more than twice its complement. Find the measure of the angle.

45. An angle’s measure is twice the measure of its complement. The larger angle is how many degrees greater than the smaller angle?

46. The supplement of an angle is 36° less than twice the supplement of the complement of the angle. Find the measure of the supplement.

**SPIRAL REVIEW**

Solve each equation. Check your answer. (Previous course)

47. \( 4x + 10 = 42 \)

48. \( 5m - 9 = m + 4 \)

49. \( 2(y + 3) = 12 \)

50. \( -(d + 4) = 18 \)

\( Y \) is between \( X \) and \( Z \), \( XY = 3x + 1 \), \( YZ = 2x - 2 \), and \( XZ = 84 \). Find each of the following. (Lesson 1-2)

51. \( x \)

52. \( XY \)

53. \( YZ \)

\( \overrightarrow{XY} \) bisects \( \angle WYZ \). Given \( m\angle WXY = 26° \), find each of the following. (Lesson 1-3)

54. \( m\angle XYZ \)

55. \( m\angle WYZ \)
Can You Dig It? A group of college and high school students participated in an archaeological dig. The team discovered four fossils. To organize their search, Sierra used a protractor and ruler to make a diagram of where different members of the group found fossils. She drew the locations based on the location of the campsite. The campsite is located at $X$ on $XB$. The four fossils were found at $R$, $T$, $W$, and $M$.

1. Are the locations of the campsite at $X$ and the fossils at $R$ and $T$ collinear or noncollinear?

2. How is $X$ related to $RT$? If $RX = 10x - 6$ and $XT = 3x + 8$, what is the distance between the locations of the fossils at $R$ and $T$?

3. $\angle RXB$ and $\angle BXT$ are right angles. Find the measure of each angle formed by the locations of the fossils and the campsite. Then classify each angle by its measure.

4. Identify the special angle pairs shown in the diagram of the archaeological dig.
Quiz for Lessons 1-1 Through 1-4

1-1 Understanding Points, Lines, and Planes

Draw and label each of the following.

1. a segment with endpoints X and Y
2. a ray with endpoint M that passes through P
3. three coplanar lines intersecting at a point
4. two points and a line that lie in a plane

Use the figure to name each of the following.

5. three coplanar points
6. two lines
7. a plane containing T, V, and X
8. a line containing V and Z

1-2 Measuring and Constructing Segments

Find the length of each segment.

9. SV
10. TR
11. ST

12. The diagram represents a straight highway with three towns, Henri, Joaquin, and Kenard. Find the distance from Henri H to Joaquin J.

13. Sketch, draw, and construct a segment congruent to CD.
14. Q is the midpoint of PR, PQ = 2z, and PR = 8z – 12. Find z, PQ, and PR.

1-3 Measuring and Constructing Angles

15. Name all the angles in the diagram.

Classify each angle by its measure.

16. m∠PVQ = 21°
17. m∠RVT = 96°
18. m∠PVS = 143°
19. RS bisects ∠QRT, m∠QRS = (3x + 8)°, and m∠SRT = (9x – 4)°. Find m∠SRT.
20. Use a protractor and straightedge to draw a 130° angle. Then bisect the angle.

1-4 Pairs of Angles

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

21. ∠1 and ∠2
22. ∠4 and ∠5
23. ∠3 and ∠4

If m∠T = (5x – 10)°, find the measure of each of the following.

24. supplement of ∠T
25. complement of ∠T
OBJECTIVE
Apply formulas for perimeter, area, and circumference.

VOCABULARY
perimeter
area
base
height
diameter
radius
circumference
pi

Why learn this?
Puzzles use geometric-shaped pieces. Formulas help determine the amount of materials needed. (See Exercise 6.)

The **perimeter** $P$ of a plane figure is the sum of the side lengths of the figure. The **area** $A$ of a plane figure is the number of nonoverlapping square units of a given size that exactly cover the figure.

\[
\text{area} = 2 \text{ units} \times 5 \text{ units} = 10 \text{ square units}
\]

Perimeter and Area

<table>
<thead>
<tr>
<th>RECTANGLE</th>
<th>SQUARE</th>
<th>TRIANGLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P = 2\ell + 2w$ or $2(\ell + w)$</td>
<td>$P = 4s$</td>
<td>$P = a + b + c$</td>
</tr>
<tr>
<td>$A = \ell w$</td>
<td>$A = s^2$</td>
<td>$A = \frac{1}{2}bh$ or $\frac{bh}{2}$</td>
</tr>
</tbody>
</table>

The **base** $b$ can be any side of a triangle. The **height** $h$ is a segment from a vertex that forms a right angle with a line containing the base. The height may be a side of the triangle or in the interior or the exterior of the triangle.

**EXAMPLE 1**

Find the perimeter and area of each figure.

A. rectangle in which $\ell = 17$ cm and $w = 5$ cm

$P = 2\ell + 2w$

$= 2(17) + 2(5)$

$= 34 + 10 = 44$ cm

$A = \ell w$

$= (17)(5) = 85$ cm$^2$

B. triangle in which $a = 8$, $b = (x + 1)$, $c = 4x$, and $h = 6$

$P = a + b + c$

$= 8 + (x + 1) + 4x$

$= 5x + 9$

$A = \frac{1}{2}bh$

$= \frac{1}{2}(x + 1)(6) = 3x + 3$

1. Find the perimeter and area of a square with $s = 3.5$ in.
**Example 2**

**Crafts Application**

The Texas Treasures quilt block includes 24 purple triangles. The base and height of each triangle are about 3 in. Find the approximate amount of fabric used to make the 24 triangles.

The area of one triangle is

\[ A = \frac{1}{2}bh = \frac{1}{2}(3)(3) = 4.5 \text{ in}^2. \]

The total area of the 24 triangles is

\[ 24 \left( \frac{4.5}{2} \right) = 108 \text{ in}^2. \]

2. Find the amount of fabric used to make the four rectangles. Each rectangle has a length of 6\(\frac{1}{2}\) in. and a width of 2\(\frac{1}{2}\) in.

In a circle a **diameter** is a segment that passes through the center of the circle and whose endpoints are on the circle. A **radius** of a circle is a segment whose endpoints are the center of the circle and a point on the circle. The **circumference** of a circle is the distance around the circle.

**Circumference and Area of a Circle**

The circumference \(C\) of a circle is given by the formula \(C = \pi d\) or \(C = 2\pi r\).

The area \(A\) of a circle is given by the formula \(A = \pi r^2\).

The ratio of a circle's circumference to its diameter is the same for all circles. This ratio is represented by the Greek letter \(\pi\) (pi). The value of \(\pi\) is irrational. Pi is often approximated as 3.14 or \(\frac{22}{7}\).

**Example 3**

**Finding the Circumference and Area of a Circle**

Find the circumference and area of the circle.

\[
\begin{align*}
C &= 2\pi r \\
&= 2\pi(3) = 6\pi \\
&\approx 18.8 \text{ cm}
\end{align*}
\]

\[
\begin{align*}
A &= \pi r^2 \\
&= \pi(3)^2 = 9\pi \\
&\approx 28.3 \text{ cm}^2
\end{align*}
\]

3. Find the circumference and area of a circle with radius 14 m.

**Think and Discuss**

1. Describe three different figures whose areas are each 16 in\(^2\).

2. **GET ORGANIZED** Copy and complete the graphic organizer. In each shape, write the formula for its area and perimeter.
**GUIDED PRACTICE**

**Vocabulary** Apply the vocabulary from this lesson to answer each question.

1. Explain how the concepts of **perimeter** and **circumference** are related.

2. For a rectangle, length and width are sometimes used in place of __?___.
   (base and height or radius and diameter)

---

**SEE EXAMPLE 1**

Find the perimeter and area of each figure.

3. [Diagram: rectangle with dimensions 11 mm by 4 mm]

4. [Diagram: square with side length 3 units]

5. [Diagram: triangle with base 5 m, height 4 m, and side 13 m]

---

**SEE EXAMPLE 2**

A puzzle contains a triangular piece with a base of 3 in. and a height of 4 in. A manufacturer wants to make 80 puzzles. Find the amount of wood used if each puzzle contains 20 triangular pieces.

---

**SEE EXAMPLE 3**

Find the circumference and area of each circle. Use the π key on your calculator. Round to the nearest tenth.

6. [Diagram: circle with radius 2.1 m]

7. [Diagram: circle with radius 7 in.]

8. [Diagram: circle with radius 16 cm]

---

**PRACTICE AND PROBLEM SOLVING**

Find the perimeter and area of each figure.

9. [Diagram: rectangle with dimensions 7.4 m by x + 6]

10. [Diagram: square with side length 5x]

11. [Diagram: triangle with sides 4x, 3x, and 8]

12. [Diagram: triangle with sides 5x, 4x, and 3x]

13. **Crafts** The quilt pattern includes 32 small triangles. Each has a base of 3 in. and a height of 1.5 in. Find the amount of fabric used to make the 32 triangles.

Find the circumference and area of each circle with the given radius or diameter. Use the π key on your calculator. Round to the nearest tenth.

14. \( r = 12 \text{ m} \)

15. \( d = 12.5 \text{ ft} \)

16. \( d = \frac{1}{2} \text{ mi} \)

Find the area of each of the following.

17. square whose sides are 9.1 yd in length

18. square whose sides are \((x + 1)\) in length

19. triangle whose base is \(5 \frac{1}{2}\) in. and whose height is \(2 \frac{1}{4}\) in.
Given the area of each of the following figures, find each unknown measure.

20. The area of a triangle is 6.75 m$^2$. If the base of the triangle is 3 m, what is the height of the triangle?

21. A rectangle has an area of 347.13 cm$^2$. If the length is 20.3 cm, what is the width of the rectangle?

22. The area of a circle is 64$\pi$. Find the radius of the circle.

23. **ERROR ANALYSIS** Below are two statements about the area of the circle. Which is incorrect? Explain the error.

<table>
<thead>
<tr>
<th>A</th>
<th>$A = \pi r^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$= \pi (8)^2$</td>
</tr>
<tr>
<td></td>
<td>$= 64\pi$ cm$^2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B</th>
<th>$A = \pi r^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$= \pi (4)^2$</td>
</tr>
<tr>
<td></td>
<td>$= 16\pi$ cm$^2$</td>
</tr>
</tbody>
</table>

Find the area of each circle. Leave answers in terms of $\pi$.

24. circle with a diameter of 28 m

25. circle with a radius of $3y$

26. **Geography** The radius $r$ of the earth at the equator is approximately 3964 mi. Find the distance around the earth at the equator. Use the $\pi$ key on your calculator and round to the nearest mile.

27. **Critical Thinking** Explain how the formulas for the perimeter and area of a square may be derived from the corresponding formulas for a rectangle.

28. Find the perimeter and area of a rectangle whose length is $(x + 1)$ and whose width is $(x - 3)$. Express your answer in terms of $x$.

29. **Multi-Step** If the height $h$ of a triangle is 3 inches less than the length of the base $b$, and the area $A$ of the triangle is 19 times the length of the base, find $b$ and $h$.

30. This problem will prepare you for the Multi-Step Test Prep on page 58.
A landscaper is to install edging around a garden. The edging costs $1.39 for each 24-inch-long strip. The landscaper estimates it will take 4 hours to install the edging.

- a. If the total cost is $120.30, what is the cost of the material purchased?
- b. What is the charge for labor?
- c. What is the area of the semicircle to the nearest tenth?
- d. What is the area of each triangle?
- e. What is the total area of the garden to the nearest foot?
31. **Algebra** The large rectangle has length $a + b$ and width $c + d$. Therefore, its area is $(a + b)(c + d)$.

   a. Find the area of each of the four small rectangles in the figure. Then find the sum of these areas. Explain why this sum must be equal to the product $(a + b)(c + d)$.

   b. Suppose $b = d = 1$. Write the area of the large rectangle as a product of its length and width. Then find the sum of the areas of the four small rectangles. Explain why this sum must be equal to the product $(a + 1)(c + 1)$.

   c. Suppose $b = d = 1$ and $a = c$. Write the area of the large rectangle as a product of its length and width. Then find the sum of the areas of the four small rectangles. Explain why this sum must be equal to the product $(a + 1)^2$.

32. **Sports** The table shows the minimum and maximum dimensions for rectangular soccer fields used in international matches. Find the difference in area of the largest possible field and the smallest possible field.

<table>
<thead>
<tr>
<th></th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>100 m</td>
<td>110 m</td>
</tr>
<tr>
<td>Width</td>
<td>64 m</td>
<td>75 m</td>
</tr>
</tbody>
</table>

Find the value of each missing measure of a triangle.

33. $b = 2$ ft; $h = \underline{3}$ ft; $A = 28$ ft$^2$

34. $b = \underline{5}$ ft; $h = 22.6$ yd; $A = 282.5$ yd$^2$

Find the area of each rectangle with the given base and height.

35. 9.8 ft; 2.7 ft

36. 4 mi 960 ft; 440 ft

37. 3 yd 12 ft; 11 ft

Find the perimeter of each rectangle with the given base and height.

38. 21.4 in.; 7.8 in.

39. 4 ft 6 in.; 6 in.

40. 2 yd 8 ft; 6 ft

Find the diameter of the circle with the given measurement. Leave answers in terms of $\pi$.

41. $C = 14$

42. $A = 100\pi$

43. $C = 50\pi$

44. A skate park consists of a two adjacent rectangular regions as shown. Find the perimeter and area of the park.

45. **Critical Thinking** Explain how you would measure a triangular piece of paper if you wanted to find its area.

46. **Write About It** A student wrote in her journal, “To find the perimeter of a rectangle, add the length and width together and then double this value.” Does her method work? Explain.
49. Which equation best represents the area $A$ of the triangle?

- A $A = 2x^2 + 4x$
- B $A = 4x(x + 2)$
- C $A = 2x^2 + 2$
- D $A = 4x^2 + 8$

50. Ryan has a 30 ft piece of string. He wants to use the string to lay out the boundary of a new flower bed in his garden. Which of these shapes would use all the string?

- F A circle with a radius of about 37.2 in.
- G A rectangle with a length of 6 ft and a width of 5 ft
- H A triangle with each side 9 ft long
- J A square with each side 90 in. long

**Challenge and Extend**

51. A circle with a 6 in. diameter is stamped out of a rectangular piece of metal as shown. Find the area of the remaining piece of metal. Use the $\pi$ key on your calculator and round to the nearest tenth.

52. a. Solve $P = 2\ell + 2w$ for $w$.
   b. Use your result from part a to find the width of a rectangle that has a perimeter of 9 ft and a length of 3 ft.

53. Find all possible areas of a rectangle whose sides are natural numbers and whose perimeter is 12.

54. **Estimation** The Ahmes Papyrus dates from approximately 1650 B.C.E. Lacking a precise value for $\pi$, the author assumed that the area of a circle with a diameter of 9 units had the same area as a square with a side length of 8 units. By what percent did the author overestimate or underestimate the actual area of the circle?

55. **Multi-Step** The width of a painting is $\frac{3}{5}$ the measure of the length of the painting. If the area is 320 in$^2$, what are the length and width of the painting?

**Spiral Review**

Determine the domain and range of each function. *(Previous course)*

56. $\{(2, 4), (-5, 8), (-3, 4)\}$
57. $\{(4, -2), (-2, 8), (16, 0)\}$

Name the geometric figure that each item suggests. *(Lesson 1-1)*

58. the wall of a classroom
59. the place where two walls meet

60. Marion has a piece of fabric that is 10 yd long. She wants to cut it into 2 pieces so that one piece is 4 times as long as the other. Find the lengths of the two pieces. *(Lesson 1-2)*

61. Suppose that $A$, $B$, and $C$ are collinear points. $B$ is the midpoint of $AC$. The coordinate of $A$ is $-8$, and the coordinate of $B$ is $-2.5$. What is the coordinate of $C$? *(Lesson 1-2)*

62. An angle's measure is 9 degrees more than 2 times the measure of its supplement. Find the measure of the angle. *(Lesson 1-4)*
Graphing in the Coordinate Plane

The coordinate plane is used to name and locate points. Points in the coordinate plane are named by ordered pairs of the form \((x, y)\). The first number is the \(x\)-coordinate. The second number is the \(y\)-coordinate. The \(x\)-axis and \(y\)-axis intersect at the origin, forming right angles. The axes separate the coordinate plane into four regions, called quadrants, numbered with Roman numerals placed counterclockwise.

**Examples**

1. **Name the coordinates of \(P\).**
   Starting at the origin \((0, 0)\), you count 1 unit to the right. Then count 3 units up. So the coordinates of \(P\) are \((1, 3)\).

2. **Plot and label \(H(−2, −4)\) on a coordinate plane. Name the quadrant in which it is located.**
   Start at the origin \((0, 0)\) and move 2 units left. Then move 4 units down. Draw a dot and label it \(H\). \(H\) is in Quadrant III.
   You can also use a coordinate plane to locate places on a map.

**Try This**

Name the coordinates of the point where the following streets intersect.
1. Chestnut and Plum
2. Magnolia and Chestnut
3. Oak and Hawthorn
4. Plum and Cedar

Name the streets that intersect at the given points.
5. \((-3, -1)\) 6. \((4, -1)\)
7. \((1, 3)\) 8. \((-2, 1)\)
Objectives
Develop and apply the formula for midpoint.
Use the Distance Formula and the Pythagorean Theorem to find the distance between two points.

Vocabulary
coordinate plane
leg
hypotenuse

Why learn this?
You can use a coordinate plane to help you calculate distances. (See Example 5.)

Major League baseball fields are laid out according to strict guidelines. Once you know the dimensions of a field, you can use a coordinate plane to find the distance between two of the bases.

A coordinate plane is a plane that is divided into four regions by a horizontal line (x-axis) and a vertical line (y-axis). The location, or coordinates, of a point are given by an ordered pair \((x, y)\).

You can find the midpoint of a segment by using the coordinates of its endpoints. Calculate the average of the \(x\)-coordinates and the average of the \(y\)-coordinates of the endpoints.

Midpoint Formula
The midpoint \(M\) of \(AB\) with endpoints \(A(x_1, y_1)\) and \(B(x_2, y_2)\) is found by

\[
M \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).
\]

Example 1 Finding the Coordinates of a Midpoint
Find the coordinates of the midpoint of \(CD\) with endpoints \(C(-2, -1)\) and \(D(4, 2)\).

\[
M \left( \frac{-2 + 4}{2}, \frac{-1 + 2}{2} \right) = \left( \frac{2}{2}, \frac{1}{2} \right) = \left( 1, \frac{1}{2} \right)
\]

1. Find the coordinates of the midpoint of \(EF\) with endpoints \(E(-2, 3)\) and \(F(5, -3)\).
Finding the Coordinates of an Endpoint

M is the midpoint of AB. A has coordinates (2, 2), and M has coordinates (4, -3). Find the coordinates of B.

**Step 1** Let the coordinates of B equal (x, y).

**Step 2** Use the Midpoint Formula: \((4, -3) = \left(\frac{2 + x}{2}, \frac{2 + y}{2}\right)\).

**Step 3** Find the x-coordinate. Find the y-coordinate.

\[
\begin{align*}
4 &= \frac{2 + x}{2} & \text{Set the coordinates equal.} \\
2(4) &= 2\left(\frac{2 + x}{2}\right) & \text{Multiply both sides by 2.} \\
8 &= 2 + x & \text{Simplify.} \\
8 - 2 &= 2 + x - 2 & \text{Subtract 2 from both sides.} \\
6 &= x & \text{Simplify.} \\
\end{align*}
\]

The coordinates of B are (6, -8).

2. S is the midpoint of RT. R has coordinates (-6, -1), and S has coordinates (-1, 1). Find the coordinates of T.

The Ruler Postulate can be used to find the distance between two points on a number line. The Distance Formula is used to calculate the distance between two points in a coordinate plane.

**Distance Formula**

In a coordinate plane, the distance \(d\) between two points \((x_1, y_1)\) and \((x_2, y_2)\) is

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.
\]

Using the Distance Formula

Find AB and CD. Then determine if \(\overline{AB} \cong \overline{CD}\).

**Step 1** Find the coordinates of each point.

A(0, 3), B(5, 1), C(-1, 1), and D(-3, -4)

**Step 2** Use the Distance Formula.

\[
\begin{align*}
AB &= \sqrt{(5 - 0)^2 + (1 - 3)^2} \\
&= \sqrt{5^2 + (-2)^2} \\
&= \sqrt{25 + 4} \\
&= \sqrt{29} \\
CD &= \sqrt{(-3 - (-1))^2 + (-4 - 1)^2} \\
&= \sqrt{(-2)^2 + (-5)^2} \\
&= \sqrt{4 + 25} \\
&= \sqrt{29}
\end{align*}
\]

Since \(AB = CD\), \(\overline{AB} \cong \overline{CD}\).
You can also use the Pythagorean Theorem to find the distance between two points in a coordinate plane. You will learn more about the Pythagorean Theorem in Chapter 5.

In a right triangle, the two sides that form the right angle are the legs. The side across from the right angle that stretches from one leg to the other is the hypotenuse. In the diagram, \(a\) and \(b\) are the lengths of the shorter sides, or legs, of the right triangle. The longest side is called the hypotenuse and has length \(c\).

**Theorem 1-6-1**  
**Pythagorean Theorem**

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

\[a^2 + b^2 = c^2\]

---

**Example 4**  
**Finding Distances in the Coordinate Plane**

Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, from \(A\) to \(B\).

<table>
<thead>
<tr>
<th>Method 1</th>
<th>Method 2</th>
</tr>
</thead>
</table>
| Use the Distance Formula. Substitute the values for the coordinates of \(A\) and \(B\) into the Distance Formula.  
\[AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\]  
\[= \sqrt{[2 - (-2)]^2 + (-2 - 3)^2}\]  
\[= \sqrt{4^2 + (-5)^2}\]  
\[= \sqrt{16 + 25}\]  
\[= \sqrt{41}\]  
\[\approx 6.4\]  

| Use the Pythagorean Theorem. Count the units for sides \(a\) and \(b\).  
\[a = 4\] and \(b = 5\).  
\[c^2 = a^2 + b^2\]  
\[= 4^2 + 5^2\]  
\[= 16 + 25\]  
\[= 41\]  
\[c = \sqrt{41}\]  
\[c \approx 6.4\] |

---

Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, from \(R\) to \(S\).

4a. \(R(3, 2)\) and \(S(-3, -1)\)  
4b. \(R(-4, 5)\) and \(S(2, -1)\)
Example 5

Sports Application

The four bases on a baseball field form a square with 90 ft sides. When a player throws the ball from home plate to second base, what is the distance of the throw, to the nearest tenth?

Set up the field on a coordinate plane so that home plate $H$ is at the origin, first base $F$ has coordinates $(90, 0)$, second base $S$ has coordinates $(90, 90)$, and third base $T$ has coordinates $(0, 90)$.

The distance $HS$ from home plate to second base is the length of the hypotenuse of a right triangle.

$$HS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(90 - 0)^2 + (90 - 0)^2}$$

$$= \sqrt{90^2 + 90^2}$$

$$= \sqrt{8100 + 8100}$$

$$= \sqrt{16,200}$$

$$\approx 127.3 \text{ ft}$$

5. The center of the pitching mound has coordinates $(42.8, 42.8)$. When a pitcher throws the ball from the center of the mound to home plate, what is the distance of the throw, to the nearest tenth?

Think and Discuss

1. Can you exchange the coordinates $(x_1, y_1)$ and $(x_2, y_2)$ in the Midpoint Formula and still find the correct midpoint? Explain.

2. A right triangle has sides lengths of $r$, $s$, and $t$. Given that $s^2 + t^2 = r^2$, which variables represent the lengths of the legs and which variable represents the length of the hypotenuse?

3. Do you always get the same result using the Distance Formula to find distance as you do when using the Pythagorean Theorem? Explain your answer.

4. Why do you think that most cities are laid out in a rectangular grid instead of a triangular or circular grid?

5. Get Organized  Copy and complete the graphic organizer below. In each box, write a formula. Then make a sketch that will illustrate the formula.
GUIDED PRACTICE

1. **Vocabulary** The __?__ is the side of a right triangle that is directly across from the right angle. (hypotenuse or leg)

2. Find the coordinates of the midpoint of each segment.
   2.1 \( AB \) with endpoints \( A(4, -6) \) and \( B(-4, 2) \)
   2.2 \( CD \) with endpoints \( C(0, -8) \) and \( D(3, 0) \)

3. **SEE EXAMPLE 2** p. 44
   3.1 \( MN \) with endpoints \( M(12, -7) \) and \( N(-5, -2) \)

4. **SEE EXAMPLE 3** p. 45
   4.1 \(JK\) and \(FG\)
   4.2 \(JK\) and \(RS\)

5. **SEE EXAMPLE 4** p. 46
   5.1 Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, between each pair of points.
   5.2 \(A(1, -2)\) and \(B(-4, -4)\)
   5.3 \(X(-2, 7)\) and \(Y(-2, -8)\)
   5.4 \(V(2, -1)\) and \(W(-4, 8)\)

6. **SEE EXAMPLE 5** p. 46
   6.1 Architecture The plan for a rectangular living room shows electrical wiring will be run in a straight line from the entrance \(E\) to a light \(L\) at the opposite corner of the room. What is the length of the wire to the nearest tenth?

PRACTICE AND PROBLEM SOLVING

12. \( XY \) with endpoints \( X(-3, -7) \) and \( Y(-1, 1) \)
13. \( MN \) with endpoints \( M(12, -7) \) and \( N(-5, -2) \)
14. \(M\) is the midpoint of \(QR\). \(Q\) has coordinates \((-3, 5)\), and \(M\) has coordinates \((7, -9)\). Find the coordinates of \(R\).
15. \(D\) is the midpoint of \(CE\). \(E\) has coordinates \((-3, -2)\), and \(D\) has coordinates \((2\frac{1}{2}, 1)\). Find the coordinates of \(C\).

Multi-Step Find the length of the given segments and determine if they are congruent.

16. \(DE\) and \(FG\)
17. \(DE\) and \(RS\)
Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, between each pair of points.

18.  \( U(0, 1) \) and \( V(-3, -9) \)  
19.  \( M(10, -1) \) and \( N(2, -5) \)  
20.  \( P(-10, 1) \) and \( Q(5, 5) \)

21. **Consumer Application**  Televisions and computer screens are usually advertised based on the length of their diagonals. If the height of a computer screen is 11 in. and the width is 14 in., what is the length of the diagonal? Round to the nearest inch.

22. **Multi-Step**  Use the Distance Formula to order \( AB, CD, \) and \( EF \) from shortest to longest.

23. Use the Pythagorean Theorem to find the distance from \( A \) to \( E \). Round to the nearest hundredth.

24. \( X \) has coordinates \((a, 3a)\), and \( Y \) has coordinates \((-5a, 0)\). Find the coordinates of the midpoint of \((XY)\).

25. Describe a shortcut for finding the midpoint of a segment when one of its endpoints has coordinates \((a, b)\) and the other endpoint is the origin.

On the map, each square of the grid represents 1 square mile. Find each distance to the nearest tenth of a mile.

26. Find the distance along Highway 201 from Cedar City to Milltown.

27. A car breaks down on Route 1, at the midpoint between Jefferson and Milltown. A tow truck is sent out from Jefferson. How far does the truck travel to reach the car?

28. **History**  The Forbidden City in Beijing, China, is the world's largest palace complex. Surrounded by a wall and a moat, the rectangular complex is 960 m long and 750 m wide. Find the distance, to the nearest meter, from one corner of the complex to the opposite corner.

29. **Critical Thinking**  Give an example of a line segment with midpoint \((0, 0)\).

The coordinates of the vertices of \( \triangle ABC \) are \( A(1, 4) \), \( B(-2, -1) \), and \( C(-3, -2) \).

30. Find the perimeter of \( \triangle ABC \) to the nearest tenth.

31. The height \( h \) to side \( BC \) is \( \sqrt{2} \), and \( b \) is the length of \( BC \). What is the area of \( \triangle ABC \)?

32. **Write About It**  Explain why the Distance Formula is not needed to find the distance between two points that lie on a horizontal or a vertical line.

33. This problem will prepare you for the Multi-Step Test Prep on page 58. Tania uses a coordinate plane to map out plans for landscaping a rectangular patio area. On the plan, one square represents 2 feet. She plans to plant a tree at the midpoint of \( \overline{AC} \). How far from each corner of the patio does she plant the tree? Round to the nearest tenth.
34. Which segment has a length closest to 4 units?

- \( EF \)
- \( JK \)
- \( GH \)
- \( LM \)

35. Find the distance, to the nearest tenth, between the midpoints of \( LM \) and \( JK \).

- \( 1.8 \)
- \( 4.0 \)
- \( 3.6 \)
- \( 5.3 \)

36. What are the coordinates of the midpoint of a line segment that connects the points \((7, -3)\) and \((-5, 6)\)?

- \( (6, -4\frac{1}{2}) \)
- \( (2, \frac{1}{2}) \)
- \( (2, 3) \)
- \( (1, 1\frac{1}{2}) \)

37. A coordinate plane is placed over the map of a town. A library is located at \((-5, 1)\), and a museum is located at \((3, 5)\). What is the distance, to the nearest tenth, from the library to the museum?

- \( 4.5 \)
- \( 5.7 \)
- \( 6.3 \)
- \( 8.9 \)

**CHALLENGE AND EXTEND**

38. Use the diagram to find the following.

a. \( P \) is the midpoint of \( AB \), and \( R \) is the midpoint of \( BC \). Find the coordinates of \( Q \).

b. Find the area of rectangle \( PBRQ \).

c. Find \( DB \). Round to the nearest tenth.

39. The coordinates of \( X \) are \((a - 5, -2a)\). The coordinates of \( Y \) are \((a + 1, 2a)\). If the distance between \( X \) and \( Y \) is 10, find the value of \( a \).

40. Find two points on the \( y \)-axis that are a distance of 5 units from \((4, 2)\).

41. Given \( \angle ACB \) is a right angle of \( \triangle ABC \), \( AC = x \), and \( BC = y \), find \( AB \) in terms of \( x \) and \( y \).

**SPIRAL REVIEW**

Determine if the ordered pair \((-1, 4)\) satisfies each function. *(Previous course)*

42. \( y = 3x - 1 \)

43. \( f(x) = 5 - x^2 \)

44. \( g(x) = x^2 - x + 2 \)

\( BD \) bisects \( \triangle ABC \), and \( BE \) bisects \( \angle CBD \). Find the measure of each angle and classify it as acute, right, or obtuse. *(Lesson 1-3)*

45. \( \angle ABD \)

46. \( \angle CBE \)

47. \( \angle ABE \)

Find the area of each of the following. *(Lesson 1-5)*

48. square whose perimeter is 20 in.

49. triangle whose height is 2 ft and whose base is twice its height

50. rectangle whose length is \( x \) and whose width is \( 4x + 5 \)
Who uses this?
Artists use transformations to create decorative patterns. (See Example 4.)

The Alhambra, a 13th-century palace in Grenada, Spain, is famous for the geometric patterns that cover its walls and floors. To create a variety of designs, the builders based the patterns on several different transformations.

A transformation is a change in the position, size, or shape of a figure. The original figure is called the preimage. The resulting figure is called the image. A transformation maps the preimage to the image. Arrow notation (→) is used to describe a transformation, and primes (′) are used to label the image.

A reflection (or flip) is a transformation across a line, called the line of reflection. Each point and its image are the same distance from the line of reflection.

A rotation (or turn) is a transformation about a point P, called the center of rotation. Each point and its image are the same distance from P.

A translation (or slide) is a transformation in which all the points of a figure move the same distance in the same direction.

EXAMPLE 1
Identify the transformation. Then use arrow notation to describe the transformation.

The transformation cannot be a translation because each point and its image are not in the same position.

The transformation is a reflection. \( \triangle EFG \rightarrow \triangle E'F'G' \)
Identify the transformation. Then use arrow notation to describe the transformation.

The transformation is a 90° rotation. $RSTU \rightarrow R'S'T'U'$

Identify each transformation. Then use arrow notation to describe the transformation.

1a. 

1b. 

Example 2 Drawing and Identifying Transformations

A figure has vertices at $A(-1, 4), B(-1, 1),$ and $C(3, 1)$. After a transformation, the image of the figure has vertices at $A'(-1, -4), B'(-1, -1),$ and $C'(3, -1)$. Draw the preimage and image. Then identify the transformation.

Plot the points. Then use a straightedge to connect the vertices.

The transformation is a reflection across the $x$-axis because each point and its image are the same distance from the $x$-axis.

Example 3 Translations in the Coordinate Plane

Find the coordinates for the image of $\triangle ABC$ after the translation $(x, y) \rightarrow (x + 3, y - 4)$. Draw the image.

Step 1 Find the coordinates of $\triangle ABC$.

The vertices of $\triangle ABC$ are $A(-1, 1), B(-3, 3),$ and $C(-4, 0)$. 

To find coordinates for the image of a figure in a translation, add $a$ to the $x$-coordinates of the preimage and add $b$ to the $y$-coordinates of the preimage. Translations can also be described by a rule such as $(x, y) \rightarrow (x + a, y + b)$. 

Check it Out! 1. The transformation cannot be a reflection because each point and its image are not the same distance from a line of reflection. 

Check it Out! 2. A figure has vertices at $E(2, 0), F(2, -1), G(5, -1),$ and $H(5, 0)$. After a transformation, the image of the figure has vertices at $E'(0, 2), F(1, 2), G'(1, 5),$ and $H'(0, 5)$. Draw the preimage and image. Then identify the transformation.
Step 2 Apply the rule to find the vertices of the image.
\[ A'(-1 + 3, 1 - 4) = A'(2, -3) \]
\[ B'(-3 + 3, 3 - 4) = B'(0, -1) \]
\[ C'(-4 + 3, 0 - 4) = C'(-1, -4) \]

Step 3 Plot the points. Then finish drawing the image by using a straightedge to connect the vertices.

3. Find the coordinates for the image of \(JKLM\) after the translation \((x, y) \rightarrow (x - 2, y + 4)\). Draw the image.

Art History Application

The pattern shown is similar to a pattern on a wall of the Alhambra. Write a rule for the translation of square 1 to square 2.

Step 1 Choose 2 points
Choose a point \(A\) on the preimage and a corresponding point \(A'\) on the image. \(A\) has coordinates \((3, 1)\) and \(A'\) has coordinates \((1, 3)\).

Step 2 Translate
To translate \(A\) to \(A'\), 2 units are subtracted from the \(x\)-coordinate and 2 units are added to the \(y\)-coordinate. Therefore, the translation rule is \((x, y) \rightarrow (x - 2, y + 2)\).

4. Use the diagram to write a rule for the translation of square 1 to square 3.

THINK AND DISCUSS

1. Explain how to recognize a reflection when given a figure and its image.
2. GET ORGANIZED Copy and complete the graphic organizer. In each box, sketch an example of each transformation.
GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

1. Given the transformation \( \triangle XYZ \rightarrow \triangle X'Y'Z' \), name the preimage and image of the transformation.

2. The types of transformations of geometric figures in the coordinate plane can be described as a slide, a flip, or a turn. What are the other names used to identify these transformations?

3. Identify each transformation. Then use arrow notation to describe the transformation.

4. Identify each transformation. Then use arrow notation to describe the transformation.

5. A figure has vertices at \( A(-3, 2), B(-1, -1), \) and \( C(-4, -2) \). After a transformation, the image of the figure has vertices at \( A'(3, 2), B'(1, -1), \) and \( C'(4, -2) \). Draw the preimage and image. Then identify the transformation.

6. Multi-Step The coordinates of the vertices of \( \triangle DEF \) are \( D(2, 3), E(1, 1), \) and \( F(4, 0) \). Find the coordinates for the image of \( \triangle DEF \) after the translation \( (x, y) \rightarrow (x - 3, y - 2) \). Draw the preimage and image.

7. Animation In an animated film, a simple scene can be created by translating a figure against a still background. Write a rule for the translation that maps the rocket from position 1 to position 2.

PRACTICE AND PROBLEM SOLVING

Identify each transformation. Then use arrow notation to describe the transformation.

8. \( F' \rightarrow E' \)

9. \( W \rightarrow X \)

10. A figure has vertices at \( J(-2, 3), K(0, 3), L(0, 1), \) and \( M(-2, 1) \). After a transformation, the image of the figure has vertices at \( J'(2, 1), K'(4, 1), L'(4, -1), \) and \( M'(2, -1) \). Draw the preimage and image. Then identify the transformation.
11. **Multi-Step** The coordinates of the vertices of rectangle \(ABCD\) are \(A(-4, 1)\), \(B(1, 1)\), \(C(1, -2)\), and \(D(-4, -2)\). Find the coordinates for the image of rectangle \(ABCD\) after the translation \((x, y) \rightarrow (x + 3, y - 2)\). Draw the preimage and the image.

12. **Travel** Write a rule for the translation that maps the descent of the hot air balloon.

Which transformation is suggested by each of the following?

13. mountain range and its image on a lake

14. straight line path of a band marching down a street

15. wings of a butterfly

Given points \(F(3, 5), G(-1, 4), \) and \(H(5, 0)\), draw \(\triangle FGH\) and its reflection across each of the following lines.

16. the \(x\)-axis

17. the \(y\)-axis

18. Find the vertices of one of the triangles on the graph. Then use arrow notation to write a rule for translating the other three triangles.

A transformation maps \(A\) onto \(B\) and \(C\) onto \(D\).

19. Name the image of \(A\).

20. Name the preimage of \(B\).

21. Name the image of \(C\).

22. Name the preimage of \(D\).

23. Find the coordinates for the image of \(\triangle RST\) with vertices \(R(1, -4), S(-1, -1)\), and \(T(-5, 1)\) after the translation \((x, y) \rightarrow (x - 2, y - 8)\).

24. **Critical Thinking** Consider the translations \((x, y) \rightarrow (x + 5, y + 3)\) and \((x, y) \rightarrow (x + 10, y + 5)\). Compare the two translations.

Graph each figure and its image after the given translation.

25. \(\overline{MN}\) with endpoints \(M(2, 8)\) and \(N(-3, 4)\) after the translation \((x, y) \rightarrow (x + 2, y - 5)\)

26. \(\overline{KL}\) with endpoints \(K(-1, 1)\) and \(L(3, -4)\) after the translation \((x, y) \rightarrow (x - 4, y + 3)\)

27. **Write About It** Given a triangle in the coordinate plane, explain how to draw its image after the translation \((x, y) \rightarrow (x + 1, y + 1)\).

28. This problem will prepare you for the Multi-Step Test Prep on page 58. Greg wants to rearrange the triangular pattern of colored stones on his patio. What combination of transformations could he use to transform \(\triangle CAE\) to the image on the coordinate plane?
29. Which type of transformation maps △XYZ to △XYZ′?
   A. Reflection    C. Translation
   B. Rotation      D. Not here

30. △DEF has vertices at D(−4, 2), E(−3, −3), and F(1, 4). Which of these points is a vertex of the image of △DEF after the translation (x, y) → (x − 2, y + 1)?
   F. (−2, 1)       H. (−5, −2)
   G. (3, 3)        I. (−6, −1)

31. Consider the translation (1, 4) → (−2, 3). What number was added to the x-coordinate?
   A. −3       B. −1       C. 1       D. 7

32. Consider the translation (−5, −7) → (−2, −1). What number was added to the y-coordinate?
   F. −3       G. 3       H. 6       I. 8

CHALLENGE AND EXTEND

33. △RST with vertices R(−2, −2), S(−3, 1), and T(1, 1) is translated by (x, y) → (x − 1, y + 3). Then the image, △R′S′T′, is translated by (x, y) → (x + 4, y − 1), resulting in △R″S″T″.
   a. Find the coordinates for the vertices of △R″S″T″.
   b. Write a rule for a single translation that maps △RST to △R″S″T″.

34. Find the angle through which the minute hand of a clock rotates over a period of 12 minutes.

35. A triangle has vertices A(1, 0), B(5, 0), and C(2, 3). The triangle is rotated 90° counterclockwise about the origin. Draw and label the image of the triangle.

   Determine the coordinates for the reflection image of any point A(x, y) across the given line.
   36. x-axis
   37. y-axis

SPIRAL REVIEW

Use factoring to find the zeros of each function. (Previous course)

38. y = x² + 12x + 35
39. y = x² + 3x − 18
40. y = x² − 18x + 81
41. y = x² − 3x + 2

Given m∠A = 76.1°, find the measure of each of the following. (Lesson 1-4)
42. supplement of ∠A
43. complement of ∠A

Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, between each pair of points. (Lesson 1-6)
44. (2, 3) and (4, 6)
45. (−1, 4) and (0, 8)
46. (−3, 7) and (−6, −2)
47. (5, 1) and (−1, 3)
Explore Transformations

A transformation is a movement of a figure from its original position (preimage) to a new position (image). In this lab, you will use geometry software to perform transformations and explore their properties.

Activity 1

1. Construct a triangle using the segment tool. Use the text tool to label the vertices $A$, $B$, and $C$.

2. Select points $A$ and $B$ in that order. Choose Mark Vector from the Transform menu.

3. Select $\triangle ABC$ by clicking on all three segments of the triangle.

4. Choose Translate from the Transform menu, using Marked as the translation vector. What do you notice about the relationship between your preimage and its image?

5. What happens when you drag a vertex or a side of $\triangle ABC$?

Try This

For Problems 1 and 2 choose New Sketch from the File menu.

1. Construct a triangle and a segment outside the triangle. Mark this segment as a translation vector as you did in Step 2 of Activity 1. Use Step 4 of Activity 1 to translate the triangle. What happens when you drag an endpoint of the new segment?

2. Instead of translating by a marked vector, use Rectangular as the translation vector and translate by a horizontal distance of 1 cm and a vertical distance of 2 cm. Compare this method with the marked vector method. What happens when you drag a side or vertex of the triangle?

3. Select the angles and sides of the preimage and image triangles. Use the tools in the Measure menu to measure length, angle measure, perimeter, and area. What do you think is true about these two figures?
**Activity 2**


2. Select point $H$ and choose Mark Center from the Transform menu.

3. Select $\angle GHI$ by selecting points $G$, $H$, and $I$ in that order. Choose Mark Angle from the Transform menu.

4. Select the entire triangle $\triangle GHI$ by dragging a selection box around the figure.

5. Choose Rotate from the Transform menu, using *Marked Angle* as the angle of rotation.

6. What happens when you drag a vertex or a side of $\triangle GHI$?

**Try This**

For Problems 4–6 choose New Sketch from the File menu.

4. Instead of selecting an angle of the triangle as the rotation angle, draw a new angle outside of the triangle. Mark this angle. Mark $\angle GHI$ as Center and rotate the triangle. What happens when you drag one of the points that form the rotation angle?

5. Construct $\triangle QRS$, a new rotation angle, and a point $P$ not on the triangle. Mark $P$ as the center and mark the angle. Rotate the triangle. What happens when you drag $P$ outside, inside, or on the preimage triangle?

6. Instead of rotating by a marked angle, use *Fixed Angle* as the rotation method and rotate by a fixed angle measure of $30^\circ$. Compare this method with the marked angle method.

7. Using the fixed angle method of rotation, can you find an angle measure that will result in an image figure that exactly covers the preimage figure?
1. She drew diagram \( ABCDEF \) to represent the patio. Find the area and perimeter of the patio. How many paving stones would Julia need to purchase to pave the patio? If each stone costs $2.25, what is the total cost of the stones for the patio? Describe how you calculated your answer.

2. Julia plans to place a fountain at the midpoint of \( AF \). How far is the fountain from \( B, C, E, \) and \( F \)? Round to the nearest tenth.

3. Julia used a pair of paving stones to create another pattern for the patio. Describe the transformation she used to create the pattern. If she uses just one transformation, how many other patterns can she create using two stones? Draw all the possible combinations. Describe the transformation used to create each pattern.
Quiz for Lessons 1-5 Through 1-7

1-5 Using Formulas in Geometry
Find the perimeter and area of each figure.

1. \( \text{Perimeter } = 20 \text{ in.} \)
\( \text{Area } = 8 \text{ in.} \)

2. \( \text{Perimeter } = 3x - 11 \)
\( \text{Area } = 2x + 20 \)

3. \( \text{Perimeter } = 6x \)
\( \text{Area } = 3x + 2 \)

4. \( \text{Perimeter } = 14x - 2 \)
\( \text{Area } = 5x + 14 \)

5. Find the circumference and area of a circle with a radius of 6 m. Use the \( \pi \) key on your calculator and round to the nearest tenth.

1-6 Midpoint and Distance in the Coordinate Plane

6. Find the coordinates for the midpoint of \( \overline{XY} \) with endpoints \( X(-4, 6) \) and \( Y(3, 8) \).

7. \( J \) is the midpoint of \( \overline{HK} \), \( H \) has coordinates \( (6, -2) \), and \( J \) has coordinates \( (9, 3) \). Find the coordinates of \( K \).

8. Using the Distance Formula, find \( QR \) and \( ST \) to the nearest tenth. Then determine if \( QR \cong ST \).

9. Using the Distance Formula and the Pythagorean Theorem, find the distance, to the nearest tenth, from \( F(4, 3) \) to \( G(-3, -2) \).

1-7 Transformations in the Coordinate Plane
Identify the transformation. Then use arrow notation to describe the transformation.

10. \( A \to A' \)
\( B \to B' \)

11. \( Q \to Q' \)
\( R \to R' \)

12. A graphic designer used the translation \( (x, y) \to (x - 3, y + 2) \) to transform square \( HJKL \). Find the coordinates and graph the image of square \( HJKL \).

13. A figure has vertices at \( X(1, 1) \), \( Y(3, 1) \), and \( Z(3, 4) \). After a transformation, the image of the figure has vertices at \( X'(-1, -1) \), \( Y'(-3, -1) \), and \( Z'(-3, -4) \). Graph the preimage and image. Then identify the transformation.
Vocabulary

acute angle .................. 21  
diameter .......................... 37  
adjacent angles ............... 28  
distance ............................ 13  
angle ................................. 20  
endpoint .......................... 7  
angle bisector ..................... 23  
exterior of an angle .......... 20  
area .................................. 36  
height ............................... 36  
base .................................. 36  
hypotenuse ......................... 45  
between ............................. 14  
bisect ................................. 15  
circumference ...................... 37  
interior of an angle ............ 20  
complementary angles ........ 29  
leg ..................................... 45  
congruent angles ............... 22  
line .................................. 6  
congruent segments ............ 13  
linear pair .......................... 28  
construction ....................... 14  
measure ............................. 20  
coordinate .......................... 13  
midpoint ............................. 15  
coordinate plane .................. 43  
obtuse angle ......................... 21  
coplanar ............................ 6  
opposite rays ........................ 7  
degree ............................... 20  
perimeter ........................... 36  
distance .............................. 13  
endpoint ........................... 7  
endpoint ........................... 7  
segment ............................. 7  
segment bisector .................. 16  
segment ............................. 7  
straight angle ...................... 21  
supplementary angles .......... 29  
translation .......................... 50  
translation .......................... 50  
transformation ...................... 50  
undefined term ...................... 6  
vertex ................................. 20  
vertical angles ...................... 30

Complete the sentences below with vocabulary words from the list above.
1. A(n) ______________ divides an angle into two congruent angles.
2. ______________ are two angles whose measures have a sum of 90°.
3. The length of the longest side of a right triangle is called the ______________.

1-1 Understanding Points, Lines, and Planes (pp. 6–11)

EXAMPLES

- Name the common endpoint of $\overrightarrow{SR}$ and $\overrightarrow{ST}$.

$\overrightarrow{SR}$ and $\overrightarrow{ST}$ are opposite rays with common endpoint $S$.

EXERCISES

Name each of the following.

4. four coplanar points
5. line containing $B$ and $C$
6. plane that contains $A$, $G$, and $E$
Draw and label three coplanar lines intersecting in one point.

1-2 Measuring and Constructing Segments (pp. 13–19)

**EXAMPLES**

- Find the length of $XY$.
  \[
  XY = | -2 - 1 | = | -3 | = 3
  \]

- $S$ is between $R$ and $T$. Find $RT$.
  \[
  RT = RS + ST
  \]
  \[
  3x + 2 = 5x - 6 + 2x
  \]
  \[
  x = 2
  \]
  \[
  RT = 3(2) + 2 = 8
  \]

3-1 Measuring and Constructing Angles (pp. 20–27)

**EXAMPLES**

- Classify each angle as acute, right, or obtuse.
  \[
  \angle ABC \text{ acute};
  \angle CBD \text{ acute};
  \angle ABD \text{ obtuse};
  \angle DBE \text{ acute};
  \angle CBE \text{ obtuse}
  \]

- $KM$ bisects $\angle JKL$, $m\angle JKM = (3x + 4)^\circ$, and $m\angle MKL = (6x - 5)^\circ$. Find $m\angle JKL$.
  \[
  3x + 4 = 6x - 5
  \]
  \[
  3x + 9 = 6x
  \]
  \[
  9 = 3x
  \]
  \[
  x = 3
  \]
  \[
  m\angle JKL = 3x + 4 + 6x - 5
  \]
  \[
  = 9x - 1
  \]
  \[
  = 9(3) - 1 = 26^\circ
  \]

**EXERCISES**

- Find the length of $XY$.
  \[
  XY = \left| -2 - 1 \right| = \left| -3 \right| = 3
  \]

- $S$ is between $R$ and $T$. Find $RT$.
  \[
  RT = RS + ST
  \]
  \[
  3x + 2 = 5x - 6 + 2x
  \]
  \[
  x = 2
  \]
  \[
  RT = 3(2) + 2 = 8
  \]

- Classify each angle as acute, right, or obtuse.
  \[
  \angle ABC \text{ acute};
  \angle CBD \text{ acute};
  \angle ABD \text{ obtuse};
  \angle DBE \text{ acute};
  \angle CBE \text{ obtuse}
  \]

- $KM$ bisects $\angle JKL$, $m\angle JKM = (3x + 4)^\circ$, and $m\angle MKL = (6x - 5)^\circ$. Find $m\angle JKL$.
  \[
  3x + 4 = 6x - 5
  \]
  \[
  3x + 9 = 6x
  \]
  \[
  9 = 3x
  \]
  \[
  x = 3
  \]
  \[
  m\angle JKL = 3x + 4 + 6x - 5
  \]
  \[
  = 9x - 1
  \]
  \[
  = 9(3) - 1 = 26^\circ
  \]

- Find each length.
  \[
  JL
  \]
  \[
  HK
  \]

- $Y$ is between $X$ and $Z$, $XY = 13.8$, and $XZ = 21.4$. Find $YZ$.

- $Q$ is between $P$ and $R$. Find $PR$.

- $U$ is the midpoint of $TV$, $TU = 3x + 4$, and $UV = 5x - 2$. Find $TU$, $UV$, and $TV$.

- $E$ is the midpoint of $DF$, $DE = 9x$, and $EF = 4x + 10$. Find $DE$, $EF$, and $DF$.
1-4 Pairs of Angles (pp. 28–33)

**Examples**

- Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.
  - \( \angle 1 \) and \( \angle 2 \) are only adjacent.
  - \( \angle 2 \) and \( \angle 4 \) are not adjacent.
  - \( \angle 2 \) and \( \angle 3 \) are adjacent and form a linear pair.
  - \( \angle 1 \) and \( \angle 4 \) are adjacent and form a linear pair.

- Find the measure of the complement and supplement of each angle.
  
  \[
  90 - 67.3 = 22.7^\circ \\
  180 - 67.3 = 112.7^\circ \\
  90 - (3x - 8) = (98 - 3x)^\circ \\
  180 - (3x - 8) = (188 - 3x)^\circ 
  \]

19. \( \angle 1 \) and \( \angle 2 \)
20. \( \angle 3 \) and \( \angle 4 \)
21. \( \angle 2 \) and \( \angle 5 \)

**Exercises**

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

19. \( \angle 1 \) and \( \angle 2 \)
20. \( \angle 3 \) and \( \angle 4 \)
21. \( \angle 2 \) and \( \angle 5 \)

Find the measure of the complement and supplement of each angle.

22. \( \angle \) \( \angle \) 74.6°
23. \( \angle \) \( \angle \) \( (2x - 4)^\circ \)
24. An angle measures 5 degrees more than 4 times its complement. Find the measure of the angle.

1-5 Using Formulas in Geometry (pp. 36–41)

**Examples**

- Find the perimeter and area of the triangle.
  
  \[
  P = 2x + 3x + 5 + 10 = 5x + 15 \\
  A = \frac{1}{2} (3x + 5)(2x) = 3x^2 + 5x
  \]

- Find the circumference and area of the circle to the nearest tenth.
  
  \[
  C = 2\pi r = 2\pi (11) = 22\pi \approx 69.1 \text{ cm} \\
  A = \pi r^2 = \pi (11)^2 = 121\pi \approx 380.1 \text{ cm}^2
  \]

**Exercises**

Find the perimeter and area of each figure.

25. \( \) \( \) \( \)
26. \( \) \( \) \( x + 4 \)
27. \( \) \( \) \( \)
28. \( \) \( \) \( \)
29. \( \) \( \) \( \)
30. \( \) \( \) \( \)
31. The area of a triangle is 102 \( \text{m}^2 \). The base of the triangle is 17 \( \text{m} \). What is the height of the triangle?
1-6 Midpoint and Distance in the Coordinate Plane (pp. 43–49)

**Examples**

- **X** is the midpoint of **CD**. **C** has coordinates \((-4, 1)\), and **X** has coordinates \((3, -2)\). Find the coordinates of **D**.

  \[
  (3, -2) = \left(\frac{-4 + x}{2}, \frac{1 + y}{2}\right)
  \]

  \[
  3 = \frac{-4 + x}{2} \quad -2 = \frac{1 + y}{2}
  \]

  \[
  6 = -4 + x \quad -4 = 1 + y
  \]

  The coordinates of **D** are \((10, -5)\).

- Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, from \((-1, 6)\) to \((4, 2)\).

  \[
  d = \sqrt{(4 - 1)^2 + (2 - 6)^2}
  \]

  \[
  = \sqrt{9 + 16}
  \]

  \[
  = \sqrt{25}
  \]

  \[
  = 5.0
  \]

1-7 Transformations in the Coordinate Plane (pp. 50–55)

**Examples**

- Identify the transformation. Then use arrow notation to describe the transformation.

  ![Diagram of a triangle transformation](image)

  The transformation is a reflection. \(\triangle ABC \rightarrow \triangle A'B'C'\)

- The coordinates of the vertices of rectangle **HIJK** are **H**\((2, -1)\), **I**\((5, -1)\), **K**\((5, -3)\), and **L**\((2, -3)\). Find the coordinates of the image of rectangle **HIJK** after the translation \((x, y) \rightarrow (x - 4, y + 1)\).

  \[
  H' = (2 - 4, -1 + 1) = H'(-2, 0)
  \]

  \[
  J' = (5 - 4, -1 + 1) = J'(1, 0)
  \]

  \[
  K' = (5 - 4, -3 + 1) = K'(1, -2)
  \]

  \[
  L' = (2 - 4, -3 + 1) = L'(-2, -2)
  \]

**Exercises**

1. Identify each transformation. Then use arrow notation to describe the transformation.

2. **Y** is the midpoint of **AB**. Find the missing coordinates of each point.

   32. **A**\((3, 2)\); **B**\((-1, 4)\); **Y**\((5, \_\_\_)\)

   33. **A**\((5, 0)\); **B**\((-\_\_\_, \_\_\_)\); **Y**\((-2, 3)\)

   34. **A**\((-\_\_\_, \_\_\_)\); **B**\((-4, 4)\); **Y**\((-2, 3)\)

   Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, between each pair of points.

   35. **X**\((-2, 4)\) and **Y**\((6, 1)\)

   36. **H**\((0, 3)\) and **K**\((-2, -4)\)

   37. **L**\((-4, 2)\) and **M**\((3, -2)\)

3. The coordinates for the vertices of **XYZ** are **X**\((-5, -4)\), **Y**\((-3, -1)\), and **Z**\((-2, -2)\). Find the coordinates for the image of **XYZ** after the translation \((x, y) \rightarrow (x + 4, y + 5)\).
1. Draw and label plane \( N \) containing two lines that intersect at \( B \).

Use the figure to name each of the following.

2. four noncoplanar points
3. line containing \( B \) and \( E \)
4. The coordinate of \( A \) is \(-3\), and the coordinate of \( B \) is \(0.5\). Find \( AB \).

5. \( E, F, \) and \( G \) represent mile markers along a straight highway. Find \( EF \).

6. \( J \) is the midpoint of \( HK \). Find \( HJ, JK, \) and \( HK \).

Classify each angle by its measure.

7. \( m \angle LMP = 70^\circ \)
8. \( m \angle QMN = 90^\circ \)
9. \( m \angle PMN = 125^\circ \)
10. \( \overline{TV} \) bisects \( \angle RTS \). If the \( m \angle RTV = (16x - 6)^\circ \) and \( m \angle VTS = (13x + 9)^\circ \), what is the \( m \angle RTV \)?

11. An angle’s measure is 5 degrees less than 3 times the measure of its supplement. Find the measure of the angle and its supplement.

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

12. \( \angle 2 \) and \( \angle 3 \)
13. \( \angle 4 \) and \( \angle 5 \)
14. \( \angle 1 \) and \( \angle 4 \)

15. Find the perimeter and area of a rectangle with \( b = 8 \) ft and \( h = 4 \) ft.

Find the circumference and area of each circle to the nearest tenth.

16. \( r = 15 \) m
17. \( d = 25 \) ft
18. \( d = 2.8 \) cm

19. Find the midpoint of the segment with endpoints \((-4, 6)\) and \((3, 2)\).

20. \( M \) is the midpoint of \( LN \). \( M \) has coordinates \((-5, 1)\), and \( L \) has coordinates \((2, 4)\). Find the coordinates of \( N \).

21. Given \( A(-5, 1), B(-1, 3), C(1, 4), \) and \( D(4, 1) \), is \( \overline{AB} \equiv \overline{CD} \)? Explain.

Identify each transformation. Then use arrow notation to describe the transformation.

22. \( Q \rightarrow Q' \)
23. \( X \rightarrow X' \)

24. A designer used the translation \( (x, y) \rightarrow (x + 3, y - 3) \) to transform a triangular-shaped pin \( ABC \). Find the coordinates and draw the image of \( \triangle ABC \).
**FOCUS ON SAT**

The SAT has three sections: Math, Critical Reading, and Writing. Your SAT scores show how you compare with other students. It can be used by colleges to determine admission and to award merit-based financial aid. You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

---

1. Points $D$, $E$, $F$, and $G$ are on a line, in that order. If $DE = 2$, $FG = 5$, and $DF = 6$, what is the value of $EG(DG)$?
   - (A) 13
   - (B) 18
   - (C) 19
   - (D) 42
   - (E) 99

2. $QS$ bisects $\angle PQR$, $m\angle PQR = (4x + 2)^\circ$, and $m\angle SQR = (3x - 6)^\circ$. What is the value of $x$?
   - (A) 1
   - (B) 4
   - (C) 7
   - (D) 10
   - (E) 19

3. A rectangular garden is enclosed by a brick border. The total length of bricks used to enclose the garden is 42 meters. If the length of the garden is twice the width, what is the area of the garden?
   - (A) 7 meters
   - (B) 14 meters
   - (C) 42 meters
   - (D) 42 square meters
   - (E) 98 square meters

4. What is the area of the square?
   - (A) 16
   - (B) 25
   - (C) 32
   - (D) 36
   - (E) 41

5. If $\angle BFD$ and $\angle AFC$ are right angles and $m\angle CFD = 72^\circ$, what is the value of $x$?
   - (A) 18
   - (B) 36
   - (C) 72
   - (D) 90
   - (E) 108

---

On SAT multiple-choice questions, you receive one point for each correct answer, but you lose a fraction of a point for each incorrect response. Guess only when you can eliminate at least one of the answer choices.
Multiple Choice: Work Backward

When you do not know how to solve a multiple-choice test item, use the answer choices and work the question backward. Plug in the answer choices to see which choice makes the question true.

**Example 1**

*T* is the midpoint of \( \overline{RC} \), \( RT = 12x - 8 \), and \( TC = 28 \). What is the value of \( x \)?

- **A** -4
- **B** 2
- **C** 3
- **D** 28

Since \( T \) is the midpoint of \( \overline{RC} \), then \( RT = RC \), or \( 12x - 8 = 28 \).

Find what value of \( x \) makes the left side of the equation equal 28.

Try choice A: If \( x = -4 \), then \( 12x - 8 = 12(-4) - 8 = -56 \).
   This choice is not correct because length is always a positive number.

Try choice B: If \( x = 2 \), then \( 12x - 8 = 12(2) - 8 = 16 \).
   Since \( 16 \neq 28 \), choice B is not the answer.

Try choice C: If \( x = 3 \), then \( 12x - 8 = 12(3) - 8 = 28 \).
   Since \( 28 = 28 \), the correct answer is C, 3.

**Example 2**

Joel used 6400 feet of fencing to make a rectangular horse pen. The width of the pen is 4 times as long as the length. What is the length of the horse pen?

- **F** 25 feet
- **G** 480 feet
- **H** 640 feet
- **J** 1600 feet

Use the formula \( P = 2\ell + 2w \). \( P = 6400 \) and \( w = 4\ell \). You can work backward to determine which answer choice is the most reasonable.

Try choice J: Use mental math. If \( \ell = 1600 \), then \( 4\ell = 6400 \). This choice is not reasonable because the perimeter of the pen would then be far greater than 6400 feet.

Try choice F: Use mental math. If \( \ell = 25 \), then \( 4\ell = 100 \). This choice is incorrect because the perimeter of the pen is 6400 ft, which is far greater than \( 2(25) + 2(100) \).

Try choice H: If \( \ell = 640 \), then \( 4\ell = 2560 \). When you substitute these values into the perimeter formula, it makes a true statement.

The correct answer is H, 640 ft.
Read each test item and answer the questions that follow.

**Item A**
The measure of an angle is 3 times as great as that of its complement. Which value is the measure of the smaller angle?

- **A** 22.5°
- **C** 63.5°
- **B** 27.5°
- **D** 67.5°

1. Are there any definitions that you can use to solve this problem? If so, what are they?
2. Describe how to work backward to find the correct answer.

**Item B**
In a town's annual relay marathon race, the second runner of each team starts at mile marker 4 and runs to the halfway point of the 26-mile marathon. At that point the second runner passes the relay baton to the third runner of the team. How many total miles does the second runner of each team run?

- **F** 4 miles
- **G** 6.5 miles
- **H** 9 miles
- **I** 13 miles

3. Which answer choice should you plug in first? Why?
4. Describe, by working backward, how you know that choices F and G are not correct.

**Item C**
Consider the translation \((-2, 8) \rightarrow (8, -4)\). What number was added to the x-coordinate?

- **A** \(-12\)
- **B** \(-6\)
- **C** \(4\)
- **D** \(10\)

5. Which answer choice should you plug in first? Why?
6. Explain how to work the test question backward to determine the correct answer.

**Item D**
\(\triangle QRS\) has vertices at \(Q(3, 5), R(3, 9),\) and \(S(7, 5)\). Which of these points is a vertex of the image of \(\triangle QRS\) after the translation \((x, y) \rightarrow (x - 7, y - 6)\)?

- **F** \((-4, 3)\)
- **H** \((4, 1)\)
- **G** \((0, 0)\)
- **I** \((4, -3)\)

7. Explain how to use mental math to find an answer that is NOT reasonable.
8. Describe, by working backward, how you can determine the correct answer.

**Item E**
\(\overline{TS}\) bisects \(\angle PTR\). If \(m\angle PTS = (9x + 2)^\circ\) and \(m\angle STR = (x + 18)^\circ\), what is the value of \(x\)?

- **A** \(-10\)
- **B** \(0\)
- **C** \(2\)
- **D** \(20\)

9. Explain how to use mental math to find an answer that is NOT reasonable.
10. Describe how to use the answer choices to work backward to find which answer is reasonable.
CUMULATIVE ASSESSMENT, CHAPTER 1

Multiple Choice

Use the diagram for Items 1–3.

1. Which points are collinear?
   - A) A, B, and C
   - B) B, C, and D
   - C) A, B, and E
   - D) B, D, and E

2. What is another name for plane \( \mathbb{R} \)?
   - F) Plane \( C \)
   - H) Plane ACE
   - G) Plane \( AB \)
   - I) Plane \( BDE \)

3. Use your protractor to find the approximate measure of \( \angle ABD \).
   - A) 123°
   - B) 117°
   - C) 77°
   - D) 63°

4. \( S \) is between \( R \) and \( T \). The distance between \( R \) and \( T \) is 4 times the distance between \( S \) and \( T \). If \( RS = 18 \), what is \( RT \)?
   - F) 24
   - G) 22.5
   - H) 14.4
   - I) 6

5. A ray bisects a straight angle into two congruent angles. Which term describes each of the congruent angles that are formed?
   - A) Acute
   - B) Obtuse
   - C) Right
   - D) Straight

6. Which expression states that \( \overline{AB} \) is congruent to \( \overline{CD} \)?
   - F) \( \overline{AB} \cong \overline{CD} \)
   - H) \( \overline{AB} = \overline{CD} \)
   - G) \( \overline{AB} = \overline{CD} \)
   - I) \( \overline{AB} \cong \overline{CD} \)

7. The measure of an angle is 35°. What is the measure of its complement?
   - A) 35°
   - B) 45°
   - C) 55°
   - D) 145°

Use the diagram for Items 8–10.

8. Which of these angles is adjacent to \( \angle MQN \)?
   - F) \( \angle QMN \)
   - H) \( \angle QNP \)
   - G) \( \angle NPQ \)
   - I) \( \angle PQN \)

9. What is the area of \( \triangle NQP \)?
   - A) 3.7 square meters
   - B) 7.4 square meters
   - C) 6.8 square meters
   - D) 13.6 square meters

10. Which of the following pairs of angles are complementary?
    - F) \( \angle MNQ \) and \( \angle QNP \)
    - G) \( \angle NQP \) and \( \angle QPN \)
    - H) \( \angle MNP \) and \( \angle QNP \)
    - I) \( \angle QMN \) and \( \angle NPQ \)

11. \( K \) is the midpoint of \( \overline{JL} \). \( J \) has coordinates \((-2, -1)\), and \( K \) has coordinates \((-4, 3)\).
    What are the coordinates of \( L \)?
    - A) \((-3, -2)\)
    - B) \((-1, 1)\)
    - C) \((-10, 7)\)
    - D) \((-10, 7)\)

12. A circle with a diameter of 10 inches has a circumference equal to the perimeter of a square. To the nearest tenth, what is the length of each side of the square?
    - F) 2.5 inches
    - G) 3.9 inches
    - H) 5.6 inches
    - I) 7.9 inches

13. The map coordinates of a campground are \((1, 4)\), and the coordinates of a fishing pier are \((4, 7)\). Each unit on the map represents 1 kilometer. If Alejandro walks in a straight line from the campground to the pier, how many kilometers, to the nearest tenth, will he walk?
    - A) 3.5 kilometers
    - B) 4.2 kilometers
    - C) 6.0 kilometers
    - D) 12.1 kilometers
14. $\angle R$ is $57^\circ$. What is the measure of its supplement?
   - [F] $33^\circ$  
   - [G] $43^\circ$  
   - [H] $123^\circ$  
   - [J] $133^\circ$

15. What rule would you use to translate a triangle 4 units to the right?
   - [A] $(x, y) \rightarrow (x + 4, y)$  
   - [B] $(x, y) \rightarrow (x - 4, y)$  
   - [C] $(x, y) \rightarrow (x, y + 4)$  
   - [D] $(x, y) \rightarrow (x, y - 4)$

16. If $\angle WZ$ bisects $\angle WXY$, which of the following statements is true?
   - [F] $m\angle WZX > m\angle YWZ$  
   - [G] $m\angle WZX < m\angle YWZ$  
   - [H] $m\angle WZX = m\angle YWZ$  
   - [J] $m\angle WZX = m\angle YWZ$

17. The $x$- and $y$-axes separate the coordinate plane into four regions, called quadrants. If $(c, d)$ is a point that is not on the axes, such that $c < 0$ and $d < 0$, which quadrant would contain point $(c, d)$?
   - [A] I  
   - [B] II  
   - [C] III  
   - [D] IV

**Gridded Response**

18. The measure of $\angle 1$ is 4 times the measure of its supplement. What is the measure, in degrees, of $\angle 1$?

19. The exits for Market St. and Finch St. are 3.5 miles apart on a straight highway. The exit for King St. is at the midpoint between these two exits. How many miles apart are the King St. and Finch St. exits?

20. $R$ has coordinates $(-4, 9)$. $S$ has coordinates $(4, -6)$. What is $RS$?

21. If $\angle A$ is a supplement of $\angle B$ and is a right angle, then what is $m\angle B$ in degrees?

22. $\angle C$ and $\angle D$ are complementary. $m\angle C$ is 4 times $m\angle D$. What is $m\angle C$?

**Short Response**

23. $\triangle ABC$ has vertices $A(-2, 0), B(0, 0)$, and $C(0, 3)$. The image of $\triangle ABC$ has vertices $A(1, -4), B'(3, -4)$, and $C'(3, -1)$.
   a. Draw $\triangle ABC$ and its image $\triangle A'B'C'$ on a coordinate plane.
   b. Write a rule for the transformation of $\triangle ABC$ using arrow notation.

24. You are given the measure of $\angle 4$. You also know the following angles are supplementary: $\angle 1$ and $\angle 2$, $\angle 2$ and $\angle 3$, and $\angle 1$ and $\angle 4$.

Explain how you can determine the measures of $\angle 1$, $\angle 2$, and $\angle 3$.

25. Marian is making a circular tablecloth from a rectangular piece of fabric that measures 6 yards by 4 yards. What is the area of the largest circular piece that can be cut from the fabric? Leave your answer in terms of $\pi$. Show your work or explain in words how you found your answer.

**Extended Response**

26. Demara is creating a design using a computer illustration program. She begins by drawing the rectangle shown on the coordinate grid.

   a. Demara translates rectangle $PQRS$ using the rule $(x, y) \rightarrow (x - 4, y - 6)$. On a copy of the coordinate grid, draw this translation and label each vertex.
   b. Describe one way that Demara could have moved rectangle $PQRS$ to the same position in part a using a reflection and then a translation.
   c. On the same coordinate grid, Demara reflects rectangle $PQRS$ across the $x$-axis. She draws a figure with vertices at $(1, -3)$, $(3, -3)$, $(3, -5)$, and $(1, -5)$. Did Demara reflect rectangle $PQRS$ correctly? Explain your answer.